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AND

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

MULTIPLE-STAGE SEWAGE SLUDGE DIGESTION

By A. M. Rawn,[1] M. Am. Soc. C. E., A. Perry Banta,[2] Assoc. M. Am. Soc. C. E., and Richard Pomeroy,[3] Esq.

Synopsis

The results of a study of four years' operation of a multiple-stage sludge digestion system are presented in this paper. Satisfactory results have been obtained with a detention period of ten days. The division of the process into stages has afforded an opportunity to obtain data on the component phases of digestion. Part I includes a quantitative study of operating data. This is followed, in Part II, by a report of research on some chemical phases of sludge digestion. Gas, sludge, and sludge-liquor analyses and a discussion of chemical equilibria are included. The presence of traces of unusual compounds in sludge and sludge gas has been verified by micro-analyses.

Introduction

Two batteries of multiple-stage digestion tanks were placed in operation in July, 1931, by the Los Angeles (Calif.) County Sanitation Districts at their Joint Disposal Plant near Harbor City, Calif. The tanks were designed on basic data obtained from experimental work conducted by the Districts and incorporated certain new principles of operation and control. The principal departure from conventional design and operation is the advantage taken of a feature described as gravimetric segregation. Digesting sludge particles will tend to float or to remain in suspension due to adhering or included gas particles. As the digestion of a solid approaches completion, it will sink and remain near the bottom of the mass. Practically all inorganic particles will likewise sink, thus a combination will be found in the lower strata consisting largely of digested particles and inorganic solids.

In the design of the tanks advantage was taken of this automatic segregation or selection to separate out, and pass ahead, the lower strata to succes-

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sively lower tanks, at the same time, permitting the upper or active strata to lag and thus to utilize more fully the tank capacity for its intended purpose.

The two batteries of stage digestion tanks are identical in all their principal features. Each battery is rectangular in plan, 56 ft wide and 150 ft long, with an effective depth of about 15 ft. Three transverse walls divide each battery into four rectangular tanks, each 56 ft by 37.5 ft in plan (see Fig. 1). Each battery is built so that the bottom has a slope of 1 on 12. Thus, the sludge surfaces of successive tanks differ by 3 ft. Each tank is equipped with a hopper at its lower end, which is placed directly against the transverse wall dividing it from the next tank in succession. The tanks have concrete covers and each is equipped with two longitudinal baffle-walls extending from within a foot of the bottom, to the cover of the tank, dividing it into three bays of equal capacity.

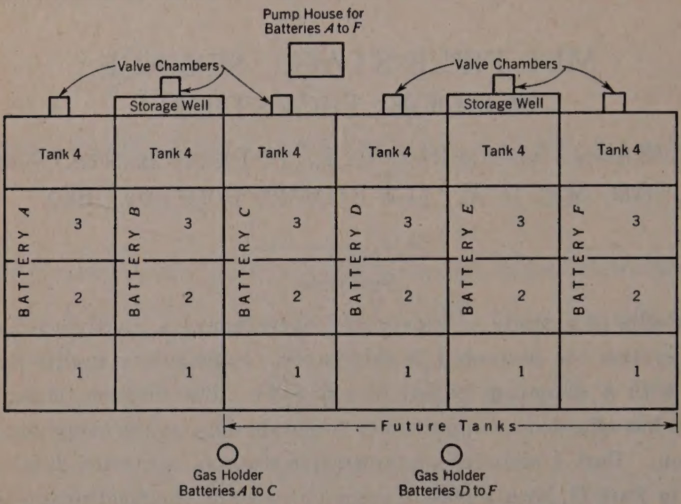
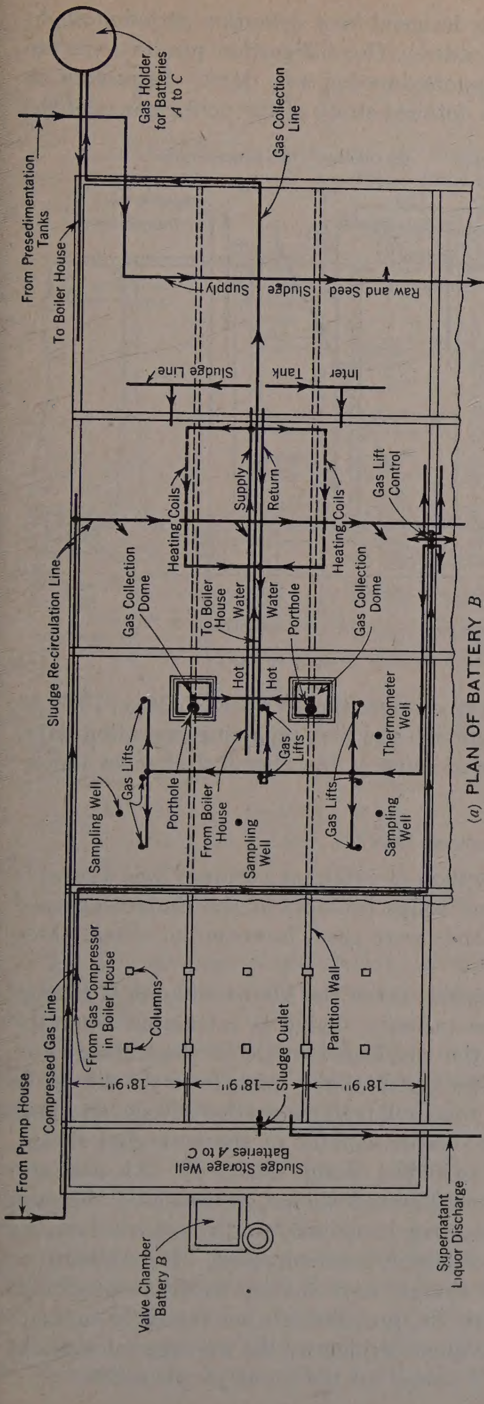


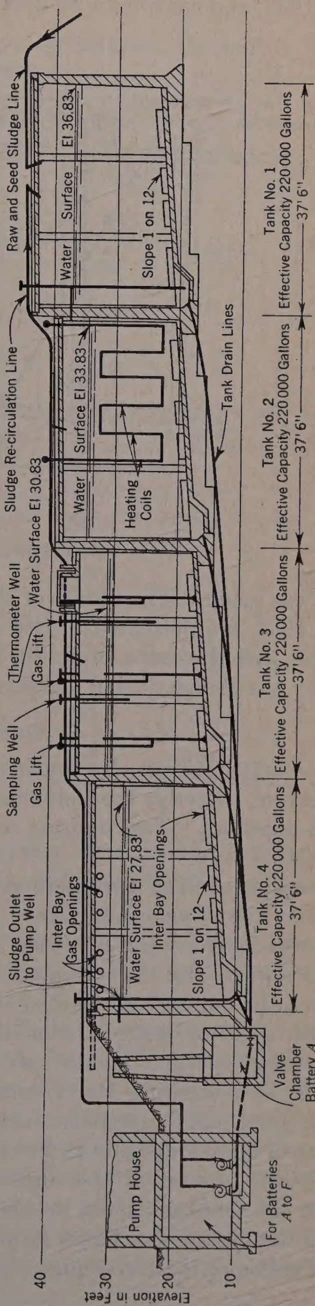
Fig. 1.—General Layout of Sludge Digestion Tanks

The process of operation is as follows (see Figs. 2 and 3): Primary and activated sludges, plus approximately 20%, by volume, of digested sludge, are introduced into the outer bays of Tank No. 1 in either of the two batteries, and the temperature of the mass is raised to approximately 85° F. To reach the central bay of Tank No. 1, solids must sink to the bottom of the mass in the outer bays and pass under the baffle-walls. To reach the discharge riser pipes, leading from Tank No. 1 to the two outer bays in Tank No. 2, solids must accumulate in the hopper at the lower end of the center bay in Tank No. 1. Thus, a process of gravimetric selection is operative in the flow of the sludge, and this selection is repeated in each successive tank until the sludge is discharged into the storage well.

An open pit was constructed adjacent to the digestion tanks which, with a capacity roughly equal to the tanks themselves, serves as a balancing reservoir during poor drying weather, promotes the removal of supernatant liquor, and incidentally adds detention time.



(a) PLAN OF BATTERY B



(b) LONGITUDINAL SECTION, BATTERY A

FIG. 2.—A TYPICAL SLUDGE DIGESTION TANK

In 1933, the tanks, which had been designed for a detention period of about twenty-five days, were heavily overloaded. Good digestion results were obtained despite a detention period approximating ten days. Beginning in April, 1933, and continuing to date, a detailed study of the performance of the tanks was made. The results of this study over the 4-yr period, May, 1933, to April, 1937, are presented in two sections—the first reporting operation data, and the second, results of research and some of the chemical changes undergone by the sludge during digestion.

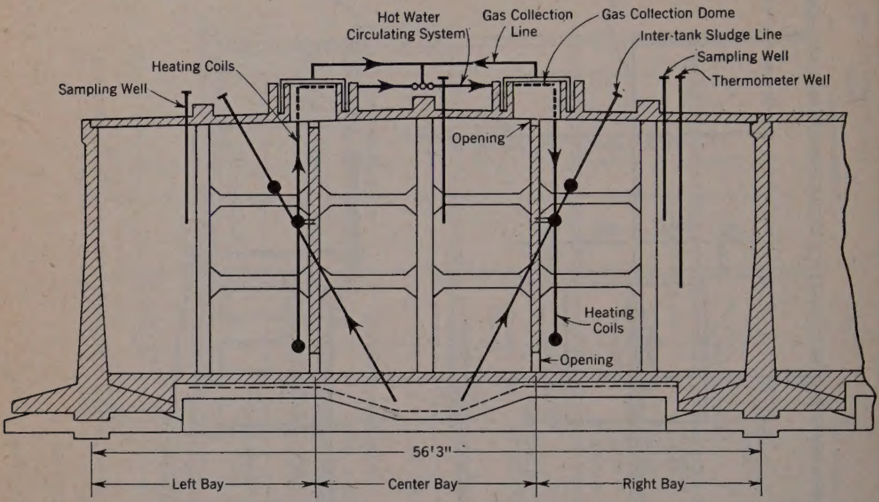


FIG. 3.—CROSS-SECTION OF A SLUDGE DIGESTION TANK

OPERATION DATA

Sludge Quantities.—No routine method of sufficient accuracy was available to measure, directly, the volumes and solids contents of the sludges pumped to the digesters. Two indirect methods were used, however, to obtain these quantities.

The first of these two methods makes use of the known volume of sewage flow and the sewage analyses. The primary sludge is calculated from the difference between the analyses of the suspended solids of the influent and settled sewage, and the activated sludge calculation is made from the difference between the non-volatile part of the total solids in the settled sewage and final effluent, together with data on the volatile content of the activated sludge. The second method makes use of digested sludge analyses. An accurate method of measuring the volume of digested sludge, recirculated digested sludge (seed), and supernatant liquor, was developed, and from such data the volume of new sludge pumped to the digesters was computed. Determinations of the ash content of the digested sludge, seed sludge, supernatant liquor, and in the sand and scum accumulations, provide data on the daily input of ash to the digesters. These results, when divided by the average ash content of the solids of the raw sludges, yield values for the input of raw solids.

Calculations of sludge quantities were made by both methods over a 2-yr period ending April 30, 1935. Since that time only the first method has been used. Results of the calculations reduced to quarterly periods are shown in Table 1.

TABLE 1.—ANALYSIS OF DATA BY QUARTERLY PERIODS

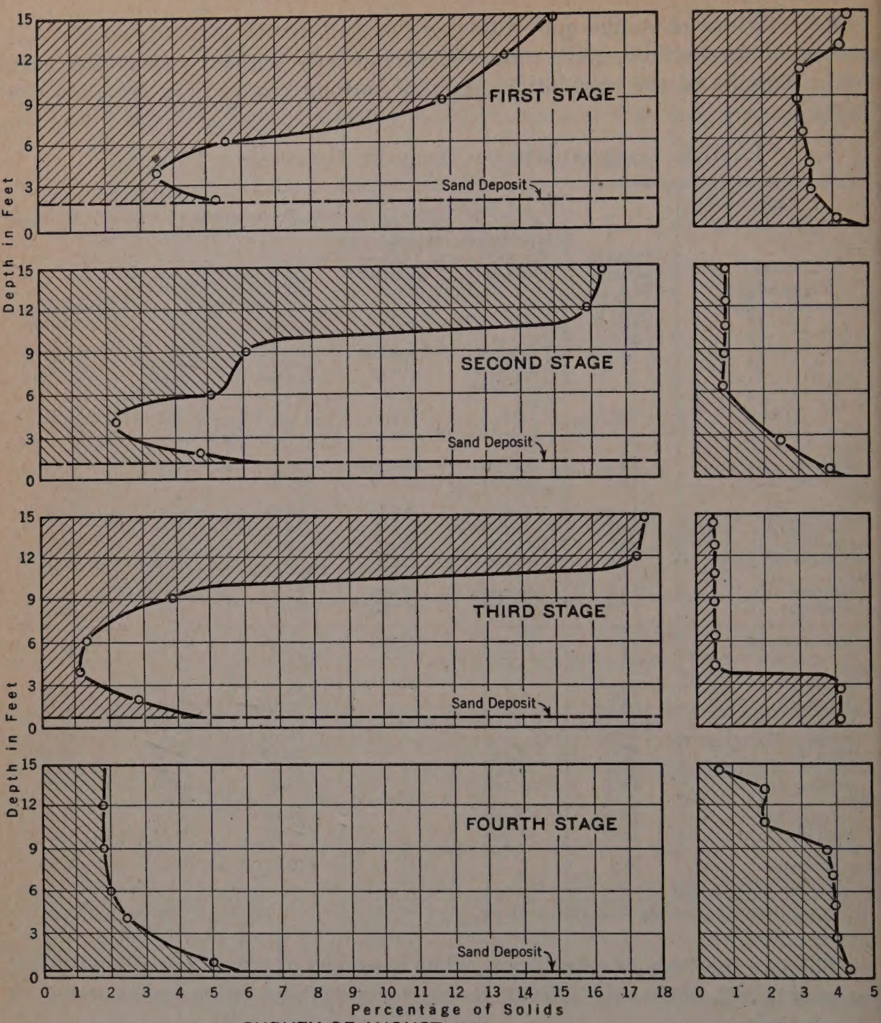
Three-month period ending:	Average sewage flow, in million gallons daily	(a) AVERAGE WEIGHT OF RAW SOLIDS PUMPED TO DIGESTERS, IN POUNDS PER DAY; COMPUTED FROM:		(b) PERCENTAGE OF ORGANIC MATTER DIGESTED IN TANKS		
				Percentage, Volatile Matter in:		Percentage digestion of organic matter
		Sewage analysis and flow	Digested sludge ash	Raw sludge	Digested sludge	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1933:						
July 31	14.1	48 700	54 600	70.61	54.66	49.8
October 31	13.7	48 300	47 800	71.48	57.24	46.6
1934:						
January 31	14.1	44 200	47 200	70.39	56.31	45.8
April 30	14.7	43 200	43 900	71.42	58.33	44.0
July 31	14.7	41 700	40 100	70.14	56.12	45.6
October 31	15.4	49 200	46 200	73.22	58.33	48.8
1935:						
January 31	15.7	50 300	48 800	71.74	57.76	46.1
April 30	15.5	45 700	47 700	67.87	53.97	44.6
2-yr. average	14.7	46 400	47 000	70.86	56.59	46.4
1935:						
July 31	16.9	....	....	68.96	54.18	46.8
October 31	18.2	....	....	70.88	56.34	46.9
1936:						
January 31	17.7	....	....	71.79	57.67	46.5
April 30	18.6	53 100	....	71.10	55.87	46.3*
July 31	18.6	51 000	....	72.58	56.22	50.6*
October 31	18.2	61 100	....	73.21	56.34	50.5*
1937:						
January 31	18.1	63 300	....	71.19	55.62	47.2*
April 30	18.6	61 000	....	67.62	52.34	44.8*
2-yr. average	18.1	....	....	70.97	55.57	47.4

* Corrected for influence of seed from the pit.

The two methods of calculation, when averaged over the 2-yr period, differ by only 1.3%, although individual quarterly periods differ by as much as 11 per cent.

The average solids content of all new sludge pumped to the digesters is 4.7 per cent. Of the new solids added to the digesters, 33% are in the primary sludge and 67% in the activated sludge.

Quarterly averages of the volume of new sludge pumped to the digesters vary from 122 000 gal per day to 160 000 gal per day. The nominal detention time (that is, the design capacity of the tanks divided by the volume of new sludge added per day) varies from 13.5 days to 10.3 days. Actually, the time of passage of the liquid component of the sludge through the tanks is about 20% less than the nominal detention time, due to the admixture of seed sludge. The time of passage of the solids may be greater or less than this fluid detention time, due to the tendency of the solids either to remain in the upper part of the tanks or to settle to the bottom and thus to pass more quickly through



(*a*) SURVEY OF AUGUST, 1933

(*b*) SURVEY OF MAY, 1934

FIG. 4

TABLE 2.— PERCENTAGES OF SOLIDS AT VARIOUS DEPTHS; TANK AVERAGES

Description	SURVEY OF AUGUST, 1933*				SURVEY OF MAY, 1934†			
	Stage 1	Stage 2	Stage 3	Stage 4	Stage 1	Stage 2	Stage 3	Stage 4
Solids.........	9.38	8.74	6.93	2.65	3.47	1.60	1.72	3.19
Volatile........	62.8	60.7	59.6	58.1	64.1	61.6	60.5	58.2

* After 25 months of continuous operation. † Seven weeks after tanks were cleaned.

the tank system; it is also increased by the occasional removal of supernatant liquor from the third stage.

Scum Accumulation.—During August, 1933, when the tanks had been in operation for 25 months, composite samples of the sludge were obtained from various elevations of each tank and were analyzed for total and organic solids. A similar survey was made of Battery A between May 11, and May 15, 1934, about seven weeks after all the tanks had been emptied and cleaned.

The percentage of organic matter tends generally to be higher at the higher elevations in the tanks, as shown in Fig. 4 and Table 2. The percentage of volatile matter for the various stages is shown in Fig. 5 and Table 2. The formation of a thick scum layer is well shown in the results of the first test, but seven weeks after the tanks had been entirely emptied there was only a slight tendency toward concentration near the sludge surface in the first tank, with no significant tendency for flotation in the succeeding tanks. The high solids content of the scum layer, together with the observed presence of seasonal seeds at least several months old, as well as quantities of match sticks, hair and bits of rubber, suggest the possibility that, after twenty-five months of operation, the scum layer was largely made up of semi-permanent, indigestible matter, and, as such, would be expected to reduce, greatly, the effective capacity of the tanks. In order to create some idea of the quantity of coarse material, samples of the scum were composited from each battery and washed on an 8-mesh screen, the retained matter being weighed. The results of these determinations are shown in Table 3. It is estimated that the screening from the scum amounted to 12% of the total weight of sludge solids in the tank.

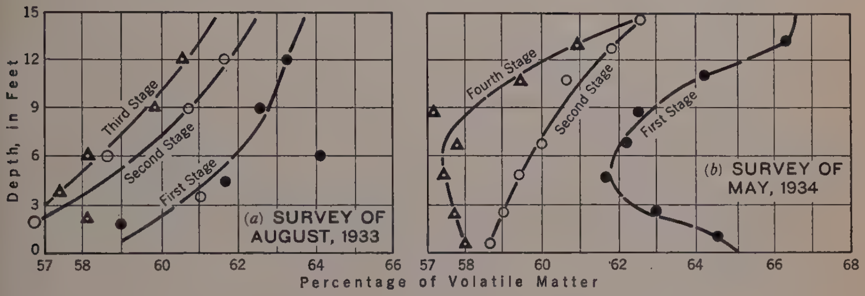


Fig. 5

TABLE 3.—Analyses of Scum

Description	First stage	Second stage	Third stage
Total solids, percentage of wet weight	16.90	16.04	16.50
Percentage of solids retained on 8-mesh screen	17.9	19.8	21.7
Percentage of organic matter in dry screening	61.8	63.8	67.7

In order to evaluate the effect of the scum accumulation on tank efficiency, experiments were conducted to determine the gas production from samples of the scum. The samples, as taken from the tanks, were placed in gallon bottles and no liquid was added, so that the conditions of tank digestion were simulated as nearly as possible. The results are shown in Table 4.

TABLE 4.—DIGESTION OF SCUM

Stage	Weight, in grams	INITIAL PERCENTAGES:		CUMULATIVE TOTAL GAS PRODUCED, IN CUBIC CENTIMETERS, ON THE FOLLOWING SUCCESSIVE DAYS:							
		Of solids	Of organic matter	1	2	3	4	5	6	7	15
1	1 027	13.97	68.6	5 140	10 420	15 580	20 730	24 020	26 140	27 460	29 760
2	1 034	16.86	62.0	2 720	4 780	6 540	7 970	9 170	....	....	....

For the first four days after removal from the tank, the scum from the first stage produced gas at the rate of 5.0 liters per day per liter of scum. This is well in excess of the averages that have been noted for any of the tanks. Even the sample from the second stage produced gas at an initial rate of 2.6 liters per liter of scum. These results show that, in spite of the accumulation of semi-permanent matter in the scum, there was, nevertheless, enough digestible sludge temporarily retained to make it the seat of very active digestion.

Reduction in Organic Matter and Gas Production.—The percentage of organic matter destroyed in the tanks is obtained from the formula:

$$\text{Fraction digested} = 1 - \frac{W_d}{W_n} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which W_d is the ratio of the weight of organic matter to the weight of ash, for digested sludge; and W_n is the corresponding ratio for new sludge.

The average for the four years ending April, 1937 (see Table 1), shows a reduction in the tanks of 47% of the organic matter added. In weight per day, this equals 17 800 lb.

Computations during the second year of the period covered by this paper showed reduction in the pit of 14% of the organic matter added to the pit; the tank system showed a reduction of 46.3% of the organic matter added, so that the tanks and the pit together effected a 53.8% reduction of the organic matter of sludge which passed through the tanks and the pit. Of the total reduction, 86% occurred in the tanks. The average detention time in the pit was 27 days. The temperature in the pit is lower than that in the tanks.

Gas measurements by stages were made during four separate periods of the first year of the tests. The limited number of meters available prevented the making of simultaneous measurements on all tanks, but single tanks were metered over 10 to 15-day periods. Table 5 gives the totals for both batteries.

In the first three periods the temperature increased in successive tanks. In the fourth period heat was applied only to the first tanks, and the temperature varied but slightly throughout the process. This increasing temperature in the first three periods of measurement probably accounts for the fact that, in two of the periods, the gas production is greater in the third stage than in the second. In the fourth period the gas production is greater in the fourth stage (when full) than in the third, probably because of the greater concentration of solids in the fourth tanks (see Fig. 2) resulting from the removal of supernatant liquor from the third tanks.

TABLE 5.—GAS PRODUCTION BY STAGES

Description	Stage 1	Stage 2	Stage 3	Stage 4*	Measured total
May 22 to June 19, 1933:					
Temperature, in degrees Fahrenheit	80.5	80.5	83.2	84.5	....
Gas Production, in:					
Cubic feet per day	109 100	63 900	65 100	51 600	264 500
Cubic feet per cubic feet of tank	1.95	1.14	1.16	0.74	1.18
August 8 to September 5, 1933:					
Temperature, in degrees Fahrenheit	80.0	83.5	85.0	85.0	....
Gas Production, in:					
Cubic feet per day	114 400	53 400	35 700	26 000	219 900
Cubic feet per cubic feet of tank	2.05	0.96	0.64	0.47	0.94
October 4 to November 2, 1933:					
Temperature, in degrees Fahrenheit	79.0	81.5	81.5	82.5	....
Gas Production, in:					
Cubic feet per day	103 800	57 000	62 800	54 500	261 500
Cubic feet per cubic feet of tank	1.86	1.02	1.12	0.98	1.17
May 5 to May 31, 1934:					
Temperature, in degrees Fahrenheit	85.6	85.3	85.3	85.3	....
Gas Production, in:					
Cubic feet per day	174 100	62 700	38 300	49 200	307 700
Cubic feet per cubic feet of tank	3.11	1.12	0.69	0.88	1.37
Average production, in cubic feet per day	125 300	59 300	50 400	45 000	263 400

* Tanks in Stage 4 were only partly filled; the values in this column were computed as for full tanks in order to permit comparison with other stages.

In the fourth period of measurement (that is, after the tanks were emptied and cleaned), the gas production in the first tanks was greater than during the other periods, which might seem to indicate an improvement resulting from the removal of the floating accumulation. This effect, however, may be accounted for adequately by two other factors, the first being the reduction of effective capacity by 13%, due to sand accumulation before cleaning, and the other being the higher temperature after cleaning.

The gas metered is slightly less than the actual production, since there is some tank leakage. On the basis of tests made when the tanks were new, the leakage under conditions of metering would be expected to be about 7 000 cu ft per day, which, added to the average daily production shown in Table 5, indicates an average daily total of 270 000 cu ft.

The gas production was measured at an average pressure of 765 mm of mercury and at 25° C. The average density of the gas under these conditions was determined experimentally to be 1.047 grams per liter. Hence, the weight of gas actually recovered was 17 400 lb per day. The average weight of organic matter destroyed over this period was 16 300 lb. Thus, the weight of gas produced was 107% of the quantity described as "weight of organic matter destroyed." This value (107%) takes no account of carbon dioxide (CO_2), ammonia (NH_3), and volatile acids lost from samples of new or digested sludge upon drying during analysis. If these losses were taken into account a still higher value would result.

In order to determine what additional gas production might be expected from longer detention times, samples of sludge from the fourth tanks were placed in bottles in the laboratory and the subsequent gas production was measured. These tests were continued until the rate of gas production had dropped to roughly one-tenth of the initial rate for the samples from the fourth tanks, or to about 2% of the rate in the first digestion tanks. In the digestion tanks the sludge yielded an average of 343 cu cm of gas per gram of solids added, or 485 cu cm per gram of organic matter added. Table 6 gives the gas yields on the same basis for the additional periods of detention.

TABLE 6.—GAS PRODUCTION AFTER SLUDGE LEAVES TANKS

Stage	Date started	Days, in detention	Average temperature, in degrees Fahrenheit	Gas Produced: In Cubic Centimeters per Gram of Original: Solids	Gas Produced: In Cubic Centimeters per Gram of Original: Organic matter	Gas Produced: Total produced, plus subsequent digestion, in cubic centimeters per gram of organic matter added	Gas Produced: Percentage of gas produced in tanks
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	August 14......	20	80.5	64	91	576	84
2	August 15......	19	80.5	60	85	570	85
3	November 1....	23	86.0	80	113	598	81
4	November 2....	22	86.0	77	108	593	82

Detailed results of the analyses of the gas will be given subsequently. The average carbon dioxide content is about 32%, the remainder being nearly pure methane gas. The content of hydrogen sulfide, H_2S, rarely exceeds 0.02 per cent. About 50 000 cu ft per day is burned in a water-heating boiler used to heat the digestion tanks. After the boiler had been in service for about eight months, it was observed that corrosion of the flues was taking place, leading to the deposition of plugs of ferrous sulfate in the lower ends of the flues. Use of a forced draft eliminated the difficulty by preventing condensation of moisture in the flues.

A 200-hp gas engine, which was installed in May, 1935, uses about 110 000 cu ft per day of the gas. The engine has run 99.3% of the time since installation, and has generated an average of 182 hp. The cost of producing this power, in terms of equivalent to electrical units, is 0.18 cts per kw-hr. This includes 10-yr amortization at 6% interest, and all operation and maintenance costs. The cylinders show no corrosion attributable to the gas. Valves are ground two or three times each year; rings were changed at the end of 23 months.

The piping of the gas-collecting system has shown no deterioration. The concrete and metal work in the digestion tanks was found to be in good condition except that in the fourth tanks the concrete above the sludge line shows moderate surface deterioration. In these tanks some air occasionally becomes mixed with the gas, and probably oxidizes hydrogen sulfide to sulfuric acid. In a few places the gas that diffused out through small cracks in the concrete has caused detectable deterioration on the exterior of the tanks where the concrete is exposed to the air. The concrete of the tops of the tanks, which are covered with a coat of tar and 12 in. of soil, is apparently unaffected, but electrical conduits laid in the soil have been practically destroyed in some places. Free sulfur has been deposited on the surface of the soil in a few small areas. It is surprising to observe these effects from such a small percentage of hydrogen sulfide in the gas.

Heating System.—The tanks are heated by means of 2-in., extra heavy, wrought-iron coils arranged vertically along the partition walls of one side of each of the first three tank compartments. Individual runs of pipe of each coil were placed in a vertical position to prevent the settlement of solids on the pipe. The coils of the first tanks were designed to supply enough heat to raise the temperature of the new sludge to the operating temperature (85° F). In the succeeding tanks only enough coil area was provided to compensate for estimated heat loss by radiation.

Soundings taken with a thermocouple indicate that there is no temperature stratification and that the tanks are quite uniform in temperature throughout the depth of each unit (see Table 7). When the tanks were emptied and cleaned after thirty months of service, approximately 0.5 in. of limey scale was removed from the heating coils. Prior to the cleaning it was necessary to use all the coils in the first three tanks or stages in order to raise the temperature to 85° F, but subsequent to the cleaning, hot water has been circulated only through the coils of the first tanks. Some minor difficulty has been experienced with air becoming trapped in the bends in the coil system. The tanks are handling more than twice the designed capacity, and additional heating surface would be desirable in the first tanks.

TABLE 7.—SLUDGE TEMPERATURES, IN DEGREES FAHRENHEIT, IN THE FIRST TANKS

Distance from top of Tank No. 1, in feet	Battery A			Battery B		
	East bay	Center bay	West bay	East bay	Center bay	West bay
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	81.7	84.0	83.7	82.8	84.4	84.7
4	....	84.0	83.6	....	....	....
7	80.9	84.1	83.6	85.2	84.2	84.7
11	....	84.0	83.7	....	....	....
15*	81.4	84.0	83.0	84.5	83.5	84.0
Average for bay	81.3	84.0	83.6	84.2	84.0	84.5
Average for tank	....	83.0	....	....	84.0	....

* * Bottom of tank.

A study of the sludge-heating data shows that 675 ft of 2-in., extra heavy, wrought-iron pipe in the first tanks transmitted heat at the rate of 84 Btu per day per ft of pipe per degree of temperature differential. This value was obtained after thirty months of service when the coils were covered with lime scale. After the scale was removed and the condition of the pipes was equivalent to new, the conductance rate was 347 Btu.

From May 15, to June 11, 1934, observations were made on the quantity of heat supplied to the digestion system. The heat transmitted daily to the tanks was 13 900 000 Btu as calculated from the sludge-heating water. New sludge at the rate of 129 000 gal per day, and at 73.2° F, was pumped to the digesters. It is calculated that 13 400 000 Btu per day was required to raise this quantity to the temperature of the first tanks. The difference of 500 000 Btu per day between the heat supplied and the heat appearing in the sludge is due: (*a*) To experimental error; (*b*) to radiation from the tanks; and (*c*) perhaps to endothermic requirements of the digestion process; or (*d*) to a combination of Causes (*a*), (*b*), and (*c*).

The difference between the free heat content of the sludge added to, and leaving, the digesters was 12 700 000 Btu per day. The head added to the sludge was 13 900 000 Btu per day; there was left 1 200 000 Btu, to account for radiation and endothermic requirements of the entire process. C. E. Keefer, M. Am. Soc. C. E., and Herman Kratz [4] found through laboratory experiments that the digestion process in general is endothermic, their experiments showing a net heat absorption varying from 154 to 433 calories per gram of organic matter destroyed. The total destruction of organic matter in the digestion tanks of the Districts, over the corresponding period, averaged 13 300 lb, or 6 030 kg per day. Using the minimum value cited by Messrs. Keefer and Kratz, this destruction should require 929 000 kg-cal, or 3 690 000 Btu, a quantity of heat three times as great as that which appears to be available (1 200 000 Btu).

A similar conclusion may be reached by comparing the temperatures in the second and fourth stages during the period when no heat was being applied beyond the first stage. On the basis of the minimum values given by Messrs. Keefer and Kratz for the endothermic requirements in the latter stages of digestion, it is calculated that the sludge temperature should fall 2.3° F even with no loss by radiation. Actually, the cooling, including that due to heat loss by radiation, was less than 0.5° F.

Foaming.—Foaming is occasionally observed in the first and second tanks. Gas space of 6 ft in the first tanks and 4 ft in the other tanks is provided to accommodate the foam, and for four years after the tanks were placed in service no trouble was encountered. In January, 1936, the foaming became serious, rising to the tops of the tanks and filling the gas lines. Seed sludge was being taken at that time from the fourth tanks. It is believed that this sludge was not far enough advanced in digestion to serve as satisfactory seed, an indication of this being the fact that the content of volatile acids was 600 ppm to 850 ppm (as acetic acid). Seed was then taken from the pit, whereupon the foaming subsided. However, the difficulty has occasionally recurred since that time although the seed is now regularly taken from the pit. It is now believed that the trouble lies in the failure to secure adequate mixing of the seed with the new sludge. It has not been convenient to pump seed to the tanks more than once each day. Between pumpings of seed, one of the first tanks may receive a volume of new sludge nearly as great as the volume of the tank. The pumping of a quantity of seed into the outside bays may not permit proper seeding of the entire mass. When the foam rises to a danger point, some of the raw sludge is by-passed to subsequent tanks until the foam subsides. This procedure has effectively prevented operating difficulty.

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[4] "The Interchange of Heat During Sludge Digestion," by Messrs. C. E. Keefer and Herman Kratz, *Sewage Works Journal*, Vol. 5 (1933), p. 3.

Stirring.—The tanks were equipped with gas lifts for the purpose of stirring the sludge, but a comparison of the performance of one battery operated with the gas lifts as against the other battery operated without stirring, failed to indicate any difference. With the degree of mixing made possible by the tank arrangement it is doubtful whether any additional stirring is warranted.

Sludge Drying.—The sludge drying beds comprised 2.34 acres of underdrained sand beds, 6.0 acres of diked field, and 2.9 acres of shallow lagoons. The diked fields are constructed by grading open fields so that the sludge depth is about 2 ft. The soil is a tight adobe that leaves little chance for seepage or under-drainage. Data indicate that sludge was dried at the rate of 663 tons per acre per yr on the sand beds and at the rate of 342 tons per acre per yr on the "field." About 25% of it was wasted directly from the digestion tanks to the drying beds. For convenience in operation the other 75% was pumped to the pit before passing to the beds or field. No difference in drying characteristics has been noted in the two sludges. Both types become dry without odor nuisance or the breeding of house flies.

Measurements of the rates of draining of sludge pumped to the sand beds gave rather erratic results, indicating considerable variation in the drainability of sludge from time to time. The most that can be concluded, from the twenty beds measured, is that, in general, when the sludge from this installation is pumped to the beds with a solids content of $4\frac{1}{2}\%$, it will lose about one-half its moisture by drainage in the first 48 hr. The remainder of the moisture is lost by evaporation.

TABLE 8.—COST OF DIGESTION, SLUDGE DRYING, AND SUPERNATANT LIQUOR FILTRATION FOR THE YEAR 1934

Description	Costs, in Dollars					Weight of Sludge, in Tons		Unit Cost, in Dollars per Ton	
	Plant cost*	Cost of Operation†							
		Power, labor, and maintenance	Supervision	Interest and depreciation‡	Total	Raw	Digested	Raw§	Digested‖
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Digestion	132 000	3 075	380	10 580	14 035	8 470	4 800	1.66	2.92
Drying on sand beds	18 500	2 460	330	1 480	4 270	2 730	1 550	1.57	2.75
Drying on fields or lagoons	3 000	500	45	250	795	5 740	3 250	0.14	0.25
Filtration of supernatant liquor	3 300	440	55	260	755	6 770	3 840	0.11	0.20

* Does not include cost of land. † Does not include cost of removing sludge from beds. ‡ 8% of Column (2). § Column (6) ÷ Column (7). ‖ Column (6) ÷ Column (8).

Costs.—Table 8 was prepared to illustrate costs for digestion and for drying on underdrained beds and open fields or lagoons. Since approximately 75%

of all the digested sludge is transferred to the pit prior to drying, the cost of the pit is included with the tank costs. It has little influence on unit costs for digestion. The cost of land on which tanks, pit, and drying areas are laid out is not included. Sludge is delivered wet, on the beds, to a fertilizer contractor who assumes the cost of its removal. The contractor pays the District $2 per ton of dry solids delivered on the beds and lagoons.

The gas engine, operated on gas from the digesters, has effected a net saving in power costs of $480 per month. No credits for this have been applied to the costs in Table 8.

Chemical Studies

Multi-stage digestion as applied in this installation affords an excellent opportunity to study, under plant-scale operation, the chemical changes occurring in the digestion process. Although some of the information presented is abstract, its inclusion is thought to be of value in helping to furnish a clearer concept of the fundamental nature of the digestion process. Basic information obtained through studies of this kind may be expected to point the way to the solution of some of the problems of sludge digestion. The following phenomena were given attention in a chemical study carried on principally during the year, May, 1933, to April 1934: Changes occurring in the sludge liquor, changes of the ether-soluble matter, nitrogen balance, and composition of the gas.

Changes in the Sludge Liquor.—Analyses were made on samples of clear liquor separated from the sludge leaving each stage of the digestion process. No complete analyses were made on liquor from the pit; but instead, two samples of sludge from the fourth tanks were retained in bottles at 86° F for an additional period of 23 days and analyzed at the end of that time. These latter samples represent approximately the results that might be expected from a total digestion time of 32 days.

Most of the analyses were made on samples taken during two test periods: August 11 to 22, 1933, and most of the month of October, 1933. No essential differences appeared between the results covering the two periods, except in the case of phosphate. Table 9 shows the average results of all valid analyses, except that the phosphate values for the two test periods are tabulated separately.

The "mixture" is computed from the known proportions by volume of new and seed sludges added to the first stage. Analyses of filtered sewage are taken as representative of the liquor accompanying the new sludge. Actually, the composition of the liquor begins to change as soon as the sludge settles from the sewage. The method of calculation has the effect of including in the change attributed to the first stage that which occurs in the sludge in the bottom of the settling tanks.

Bicarbonate values in Table 9 were computed from the ion balance, as bicarbonate titrations were not made at the time of these liquor analyses. Some subsequent work on the bicarbonate titration indicates that, in liquor from digested sludge, the computed value is less than the experimental value by an average of about 290 ppm as HCO_3. The pH-values are derived from the computed bicarbonate concentrations, the carbon dioxide content of the gas,

TABLE 9.—SUMMARY; CHEMICAL CONTENT OF SLUDGE LIQUORS, IN PARTS PER MILLION

Description	Sewage	Seed sludge	Mixture*	SAMPLE TAKEN FROM TANKS:				Content after additional detention of 23 days	Total number of individual analyses
				No. 1	No. 2	No. 3	No. 4		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Fixed solids	510	1 138	648	1 016	1 085	1 096	1 138	..	46
Volatile solids	315	1 092	486	1 815	1 687	1 122	1 092	..	46
Ammonia, as N	22	753	183	444	583	692	757	1 026	60
Organic nitrogen, N	8	116	32	148	93	64	77	113	12
Volatile acids, as acetic acid	26	177	59	829	790	470	228	57	46
Bicarbonate, HCO_3*	..	..	1 306	2 124	3 059	3 929	4 502	5 867	..
Silicon dioxide, SiO_2	32	49	35	59	103	80	100	39	12
Phosphate, PO_4	154	10	122	8	8	22	15	17	23
Sulfate, SO_4:									
August analysis	5	72	20	52	57	67	72	..	9
October analysis	6	70	20	82	61	31	24	35	15
Chloride, Cl	136	(155)	140	..	..	..	155	149	9
Calcium, Ca	73	304	124	190	251	277	282	290	25
Magnesium, Mg	22	64	31	43	51	60	56	56	24
Sodium, Na	147	164	151	..	..	166	..	153	7
Potassium, K	16	63	26	..	..	66	..	89	7
Iron, Fe	7	10	8	35	55	34	36	16	10
Hydrogen-ion concentration,* pH	..	..	..	6.74	6.93	7.06	7.14	7.26	..

* Computed.

and the ionization constant of carbonic acid.[5] The difference between computed and experimental bicarbonate values for digested sludge liquor would introduce a difference of 0.03 in the pH-value.

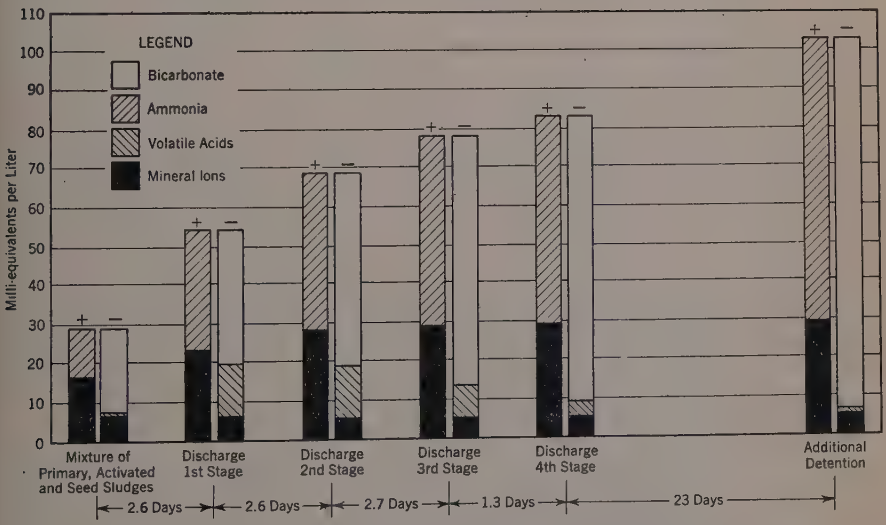

FIG. 6.—APPROXIMATE DETENTION TIME OF SLUDGE LIQUOR

Fig. 6 shows, graphically, the analyses reduced to reaction equivalents by dividing the parts per million by the reacting weights of the ions in question.

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[5] "Hydrogen Ion Concentration and Bicarbonate Equilibrium in Digesting Sludge," by A. P. Banta, Assoc. M. Am. Soc. C. E., and Richard Pomeroy, *Sewage Works Journal*, Vol. VI, 1934, p. 234.

Further consideration of some of the constituents reveals a number of interesting relationships, discussed in the following paragraphs.

Sulfate.—The rapid reduction of sulfate in the first stage is undoubtedly caused by bacterial de-oxidation with attendant liberation of sulfide ion. However, there is practically no sulfide in solution, the sulfur going to form ferrous sulfide. The apparent increase in sulfate in the later stages may have been due to chance variation of samples.

Phosphate.—Table 9 shows that in August the phosphate concentration increased regularly through the digestion tanks whereas in September the phosphate was at a maximum in the first tank and decreased progressively thereafter. It was thought that the decrease in the second period might be explained by precipitation of magnesium ammonium phosphate. This seemed likely in view of another observation. A flume that carries supernatant liquor from the pit has accumulated a crust of crystalline material, in some places to a thickness of an inch. Analyses of two samples of this material showed it to be about 96% $Mg\,NH_4\,PO_4 \cdot 6\,H_2O$.

In order to determine if this compound may precipitate in the digestion tanks, two solubility tests were made by saturating liquor from digested sludge with the pure compound, keeping the solution in equilibrium with sludge gas of known carbon dioxide content. The results yielded solubility products of of 1.06×10^{-12} at 25° C and 1.33×10^{-12} at 30° C.

On the basis of these results it is calculated that the digested liquor from the tanks, having the hydrogen-ion concentration and the content of magnesium and ammonium shown in Table 9, would be able to hold 305 ppm of phosphate in solution before becoming supersaturated. After 32 days of detention the liquor could hold 157 ppm. Hence, it does not seem possible that magnesium ammonium phosphate could precipitate in the digestion tanks, and the peculiar behavior of the phosphate in the October series of tests remains unexplained. The deposition in the flume can be explained as due to the loss of carbon dioxide and the resultant rise in pH. Such loss of carbon dioxide would be expected because of turbulence in the flume in question. If this loss allows the pH-value of the pit liquor to rise to 8.0, the proportion of phosphate that can be held is reduced to about 18 ppm (at 25° C); any excess over this concentration should precipitate.

Volatile Organic Acids.—The increase of organic acids in the first stage and their subsequent destruction supports the observations of other investigators. An attempt was made to analyze completely the volatile organic acids of sludge liquor from the first tank. A satisfactory conclusion was not reached, but by modification of an ether-extraction procedure,[6] tentative results were obtained, as follows:

Acid	Percentage
Formic	0
Acetic	63
Propionic	33
Butyric	0.1
Valeric	4

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[6] "Determination of Formic, Acetic, and Propionic Acids in a Mixture," by O. L. Osborn, H. G. Wood and C. H. Werkman, *Journal of Industrial Engineering Chemistry*, Vol. 25, 1933, p. 247.

A part of the fraction reported as valeric acid may be caproic acid. The large percentage of acetic acid undoubtedly came from decomposition of carbohydrates, whereas the presence of valeric acid suggested the de-aminization of amino acids formed by the bacterial hydrolysis of proteins. Tests for acids of the phenol group showed about 0.7 ppm (as phenol) in the liquor from the first stage, with smaller quantities in subsequent tanks.

Calcium.—The possible saturation of the sludge liquor with calcium carbonate is of some interest. In a solution saturated with calcium carbonate and in equilibrium with a gas phase containing carbon dioxide at a partial pressure, $p\ CO_2$, the following relationship should hold:

$$\frac{(Ca^{++})\,(H\,CO_3)^2}{p\ CO_2} = K \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The value of K is dependent upon the ionic strength of the solution according to the equation:

$$\log K = \log K_0 + 2\sqrt{\mu} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which μ is the ionic strength and K_0 is the value of K at zero ionic strength. The values of K corresponding to the ionic strengths of the various sludge liquors have been derived from published data.[7] Results of calculations of the concentrations of calcium necessary to saturate the various sludge liquors are shown in Table 10. In all cases the calcium found is greater than the proportion calculated to be necessary to saturate the solution. The excess is probably accounted for as supersaturation of the liquor with respect to calcium carbonate. That the liquor is supersaturated is supported by observations made when the tanks were opened for cleaning. It was found that the walls, even in the first stage, were partly covered with a hard thin deposit of lime.

TABLE 10.—SOLUBILITY OF CALCIUM CARBONATE, Ca CO_3, IN SLUDGE LIQUOR

Liquor	Ionic strength	Values of K (concentrations, in mols per liter)	Values of partial pressures, $p\ CO_2$, in atmospheres	Bicarbonate, $HC\ O_3$ in parts per million	Ca required to saturate solution in parts per million calculated	Ca, in parts per million, observed
First tank........	0.062	4.4×10^{-6}	0.345	2 124	50	190
Second tank......	0.079	5.1×10^{-6}	0.313	3 059	25	251
Third tank.......	0.088	5.6×10^{-6}	0.303	3 929	16	277
Fourth tank......	0.094	5.9×10^{-6}	0.292	4 502	13	282
Digested 32 days..	0.113	6.6×10^{-6}	0.29	5 867	8	290

It is surprising that sufficient precipitation does not occur to cause a decline of calcium concentration before the sludge leaves the tanks, but there is evidence that such a decline does occur in the later stages of prolonged digestion. This evidence appeared in the samples which were subjected to the 23 days of additional digestion. A composite of the liquor from these two samples taken at the beginning of the experiment showed 332 ppm of calcium. At the end, the two samples showed, respectively, 316 and 263 ppm. Table 9 does not show any decrease for the reason that chance variations of individual samples obscure an effect which might be revealed by averages of greater numbers of samples.

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[7] *Journal*, Am. Chemical Soc., Vol. 51, p. 2082 (1929).

Iron.—The solubility of ferrous carbonate in sludge liquor is governed by equilibrium expressions similar to those applying to calcium carbonate. On the basis of solubility data,[8] it is calculated that if the sludge liquor from the fourth stage contained 0.02 ppm of iron as ferrous ion, the solution would be saturated with ferrous carbonate. The fact that much larger concentrations prevail suggests that the iron may be in the form of un-ionized or colloidal compounds. Supersaturation with respect to ferrous carbonate may also be a factor.

In one instance when a very large percentage of ferrous iron was present in the sludge going to the digestion tanks, the green color of the iron compounds persisted throughout the digestion process. A sample of this green sludge, when treated with acid, liberated a quantity of carbon dioxide that could be accounted for only by assuming that the iron was present as ferrous carbonate.

Digestion of Ether Soluble Matter.—Determinations of ether soluble matter were made on samples of primary, activated, tank, and pit sludges. (An ether-soluble determination is made as follows: A weighed sample of dried (105° C), pulverized sludge is placed in a weighed Gooch filter; washed with hydrochloric acid (H Cl), and water; dried and re-weighed; extracted for 3 hr with ether under a reflux condenser; dried and re-weighed, the loss in weight being reported as a percentage of ether soluble material on the basis of the original dry weight. By this procedure the insoluble soaps are represented by free fatty acids in the ether extract.) All analyses but one were made on samples taken between May 15 and June 10, 1933. The results are shown in Table 11.

TABLE 11.—PERCENTAGE OF ETHER SOLUBLE MATTER IN SLUDGES

Type of sludge	Number of analyses	Maximum	Minimum	Average	Type of sludge	Number of analyses	Maximum	Minimum	Average
Primary.....	6	20.7	16.0	18.2	Digested.....	7	11.6	7.8	9.1
Activated....	6	22.7	13.9	20.3	Pit seed.....	2	8.5	6.7	7.6

Composites of the material separated by ether extraction were analyzed for various constituents. In the case of the ether soluble material taken from the digested sludge, the free acids, after being separated from the remainder and dried, changed in part to a material that could be re-dissolved neither in fat solvents nor in dilute alkali. The change probably involved oxidation and polymerization. A repetition of the extraction and drying of the part that was soluble in potassium hydroxide caused more of it to become insoluble, so that it is estimated that at least one-half the free acids in the ether soluble material of the digested sludge were of an unstable type; the true fatty acids may be only a small proportion of the total acid fraction. In view of this behavior, it seems likely that the original digested sludge contains substances which, in the original state, would be soluble in ether, but which change to insoluble forms when the sludge is dried, acid treated, and re-dried during analysis.

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[8] *Journal*, Am. Chemical Soc., Vol. 40, p. 879.

Table 12(*a*) shows the results of the analyses. Some of the acid fraction may have been phenolic acids. A qualitative test for phenolic acids in the ether extract from the digested sludge gave a strong positive result; and yet, as noted previously, only traces of phenols are present in the liquor.

TABLE 12.—ETHER-SOLUBLE MATTER

Constituent	(*a*) ANALYSES OF ETHER EXTRACTS			(*b*) DIGESTION OF ETHER-SOLUBLE MATTER		
	Primary	Activated	Digested	Original sludge, in pounds	Sludge leaving the tanks, in pounds	Percentage lost
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Total weight of ether soluble material, in pounds	....	....	....	100.0	29.3	71
Free acids*	66%	62%	26%	63.4	7.6	88
Saponifiable fats	28%	20%	28%	22.9	8.2	64
Non-saponifiable matter	6%	18%	46%	13.7	13.5	2
Phosphine (PH_3)	0.11%	0.06%	0.07%	....	....	..
Equivalent weight of acids	306	309	272	....	....	..

* Insoluble soaps in sludge.

Each 100 lb of ether soluble material entering the digestion tanks contains the quantities of various constituents shown in Table 12(*b*); the fates of these constituents are also shown. The "fatty acids" are present in the sludge in combination with calcium and magnesium as insoluble soaps. Taking into account the foregoing observations, indicating that very little true fatty acids are present in the digested sludge, it appears that the insoluble soaps are destroyed quite completely in the digestion process. The very low reduction of the non-saponifiable matter (mineral oils, etc.) is to be expected.

Calculations based upon the equivalent weights of the fatty acids and the other data as given, indicate that the quantities of acids destroyed per day is equivalent to 316 lb of calcium. An equivalent quantity of bases should be released. From the liquor analyses it is calculated that 208 lb of calcium and 33 lb of magnesium per day appear in the sludge liquor. If the magnesium is converted to the calcium equivalent and added to the value for calcium, the result is 263 lb of calcium equivalent. The agreement of this result with 316 lb from the grease analyses, is considered good in view of the many sources of error. The difference is in the direction that would be accounted for by the loss of some of the acids in the digested-sludge analysis through oxidation and polymerization, and by the precipitation of calcium carbonate in the tanks.

It was thought that phosphorous-containing fats might be a significant source of the phosphate that appears in the sludge liquor. However, calculations indicate that only 7 lb per day of phosphate would be released by the digestion of the ether soluble matter, whereas the sludge liquor analyses of August, 1933, showed 70 lb per day going into solution.

Nitrogen Balance.—Nitrogen analyses were made on samples of wet sludges during May, and June, 1933, with results as shown in Table 13(*a*). On the basis of estimates of the weights and volumes of the various sludges pumped during May and June, 1933, calculations have been made of the "through-put" of nitrogen, with results as shown in Table 13(*b*). The quantities of nitrogen entering and leaving the tanks are in good agreement, indicating that there is no appreciable loss. Since this is a rather important point, a careful experiment was performed to determine if any loss of nitrogen could be detected. A laboratory sample of a mixture of strained raw and activated sludges was seeded, and carefully sampled; and five total nitrogen determinations were made by the Kjeldahl method. The mixture was incubated for 33 days, sampled, and analyzed as before. The nitrogen balance in digestion, in parts per million of nitrogen, was found to be as follows:

TABLE 13.—NITROGEN BALANCE

Item No.	Sludge	(*a*) NITROGEN ANALYSES OF WET SLUDGE				(*b*) NITROGEN "THROUGH-PUT" FOR MAY AND JUNE, 1933		
		Number of analysis	Total Nitrogen, Percentage of Dry Weight			Dry weight, in pounds	Nitrogen Analysis	
			Maximum	Minimum	Weighted average		Percentage of dry weight	Pounds per day
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	Primary	5	2.50	2.21	2.40	21 950	2.40	527
2	Activated	11	6.51	4.30	5.21	31 770	5.21	1 655
3	Pit seed	4	6.11	5.57	5.84	10 100	5.84	590
4	Total input	..	....	....	....	....	....	2 772
5	Digested sludge	9	6.36	5.15	5.85	45 290	5.85	2 648
6	Stratified liquor	..	....	....	750*	69 000	750*	52
7	Scum and sand accumulation	..	....	....	....	650†	6.5‡	42
8	Total output and accumulation	..	....	....	....	....	....	2 742§

* 750 ppm; third tanks estimated. † Organic matter. ‡ 6.5% of organic matter (estimated).
§ Compared Items Nos. 4 and 8.

	Before digestion	After digestion
	2 089	2 157
	2 084	2 114
	2 105	2 105
	2 070	2 077
	2 054	2 086
Average	2 080	2 108
Corrected for loss of weight during digestion	2 080	2 073
Average difference from mean	15	26
Probable error of averages	7	7
Probable error of difference	10	

The average concentration of nitrogen after incubation is 28 ppm greater than in the original mixture. When a correction is made for the decrease in weight of the sludge due to loss of gas, the quantity of nitrogen in the final mixture, on the basis of the original weight, is less than in the original mixture by 7 ppm, or 0.34 per cent. On the basis of deviations of the individual determinations it is estimated that the probable error of the difference is 10 ppm; hence, the apparent loss is not significant. It is probable that if an actual loss did occur, it did not exceed 17 ppm of Kjeldahl nitrogen. These results are in accord with conclusions of Messrs. A. M. Buswell and C. S. Boruff.[9]

As the sludge originally separates from the sewage, 98.6% of the nitrogen in it is held in the solids, and 1.4% is in the form of ammonium compounds or organic nitrogen in solution. When the sludge leaves the tanks, 56.8% of the original nitrogen is still held in the solids, and 43.2% is in solution. At the end of 23 days of additional digestion, the original nitrogen still held in the solids is reduced to 41.3 per cent.

If the sludge from the tanks is dried on sand beds, two-thirds of the water originally present with the sludge will have been removed as supernatant liquor and as drainage from the beds by the time the water ceases draining from the sludge. The water remaining with the sludge will hold 14.2% of the nitrogen originally present in the sludge. Prolonged and thorough drying of the sludge removes the ammonium nitrogen almost completely, but when the sludge is dried, as is usual, to about 50% moisture, 15% of this ammonium nitrogen, and presumably all the soluble organic nitrogen, remain in the moisture of the sludge. Of the nitrogen originally present 60% will still be available for fertilizer value. If the sludge is digested for an additional 23 days and then dried to 50% moisture, 46% of the original nitrogen will still be available. Of the nitrogen fertilizer value remaining after digestion in the tanks, 23% is lost due to digestion in the pit. These considerations point to the desirability of a short digestion period when the sludge is to be used for fertilizer. It must be noted, however, that a substantial amount of digestion occurs in the sludge from the tanks during drying; this is not taken into account in the foregoing calculations.

The digestion of nitrogenous matter roughly parallels the digestion of other organic matter, with the result that the percentage of nitrogen in the solid organic matter does not show any great change. If the insoluble nitrogen is calculated as percentage of the solid organic matter, the value is 5.79% for the new sludge, 6.51% for the sludge from the tanks, and 6.09% for the sludge after 32 days of digestion. The total nitrogen in the sludge from the pit (including the soluble portion) dried on the sand beds, as calculated from the analyses of the new sludges and the analyses of the sludge liquors, should be 6.54 per cent. Samples of dried pit sludge taken in May, 1934, showed a nitrogen content corresponding to 6.49% of the organic matter (average of eight analyses from two beds). Thus, again, all the nitrogen is accounted for. The average organic content of this sludge, lowered in part by inclusion of sand, was 47.5%; hence the nitrogen was 3.08% of the dry weight of the sludge. Nitrogen analyses at other times indicate that this represents average conditions.

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[9] "The Relation Between Chemical Composition of Organic Matter and the Quality and Quantity of Gas Produced During Sludge Digestion," by Buswell and Boruff, *Sewage Works Journal*, Vol IV, 1932, p. 450.

Summary of Changes in the Solids.—If the proteins of the sludge solids are assumed to be 6.25 times the nitrogen percentage, the organic matter may be divided into ether soluble matter, protein, and cellulose and miscellaneous, and the fates of these three types of organic matter may be calculated. Results of such calculations, on the basis of 100 lb of original organic matter, are shown in Table 14.

TABLE 14.—ANALYSIS AND FATE OF ORGANIC MATTER IN SLUDGE

Sludge	ORGANIC MATTER		SOLUBLE MATTER		PROTEIN		CELLULOSE AND MISCELLANEOUS	
	Total weight, in pounds	Percentage destroyed	Total weight, in pounds	Percentage destroyed	Total weight, in pounds	Percentage destroyed	Total weight, in pounds	Percentage destroyed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
New sludge	100	....	27.6	..	36.2	..	36.2	..
Sludge from tanks	52.1	47.9	8.1	71	21.2	41	22.8	37
Sludge after thirty-two days of digestion	42.2	57.8	6.5	77	16.1	56	19.6	46

Gas Analyses.—Samples of gas from various stages of the digestion process and from the laboratory supply, a mixture from all the tanks, were analyzed for various constituents. The results, subject to certain limitations, as described subsequently, are shown in Table 15.

Carbon Dioxide.—Several analyses for carbon dioxide were made by the usual absorption method. In other cases, the density of the gas was determined and the carbon dioxide percentage was estimated from an experimentally determined correlation between density and percentage of carbon dioxide.

From the known solubility of carbon dioxide in water, it is calculated that the sludge liquor from the last tanks contains 373 ppm of free CO_2 in solution. When sludge is dried for analysis, a large additional quantity of this gas is lost by decomposition of the bicarbonates, especially ammonium bicarbonate. This loss would amount to an average of 3 030 ppm for all the liquor from the tanks. In volume, the free carbon dioxide and the bicarbonate, CO_2, that would be lost upon evaporation would amount to 30 100 cu ft per day, a quantity one-third as great as that of carbon dioxide in the gas evolved from the tanks.

Hydrogen Sulfide.—Various methods were tried for the H_2S-determination. Precipitation of lead sulfide, Pb S, from a lead acetate solution containing 2% acetic acid, with filtration and weighing of the precipitate, gave fairly consistent results; but this method is open to theoretical objections. Results considered to be more reliable were obtained by passing the gas through bromine water, and precipitating and weighing the resulting sulfate as barium sulfate.

In the discussion of iron in the sludge liquor, evidence was given showing the presence of ferrous carbonate in the sludge on at least one occasion. The iron content of the sewage at this plant is usually high due to trade wastes. The sludge contains an average of 2.46% of iron (as Fe). The efficiency with which the sulfide, formed by reduction of sulfate and decomposition of organic

TABLE 15.—GAS ANALYSES

Description	PERCENTAGE DETERMINATIONS						Pounds per day	Cubic feet per day
(1)	First stage (2)	Second stage (3)	Third stage (4)	Fourth stage (5)	Laboratory (6)	Weighted average (7)	(8)	(9)
(*a*) CARBON DIOXIDE, CO_2								
Number of determinations	14	9	8	4	3	....	....	....
Content: Maximum.....	39.7	33.2	31.5	31.4	35.4	....	....	....
Minimum......	31.7	29.0	28.7	28.4	32.5	....	....	....
Average.......	34.5	31.3	30.3	29.7	33.8	32.4	9 910	87 500
(*b*) HYDROGEN SULFIDE, H_2S								
Number of determinations	5	2	3	1	6	....	....	....
Content: Maximum.....	0.023	0.0014	0.0039	....	0.0530	....	....	....
Minimum......	0.00014	0.0001	0.00009	....	0.00058	....	....	11.6
Average.......	0.0068	0.0007	0.0026	0.0035	0.012	0.0043	1.0	....
(*c*) CARBON MONOXIDE, C O								
Number of determinations	2	....	....	2	....	....	....	....
Content: Maximum.....	0.047	....	....	0.032	....	....	....	....
Minimum......	0.015	....	....	0.017	....		....	....
Average.......	0.03	....	....	0.02	....	Not more than 0.03	5.8	80
(*d*) HYDROGEN, H_2								
Number of determinations	2	....	....	2	....	....	....	....
Content: Maximum.....	0.018	....	....	0.018	....	....	....	....
Minimum......	0.018	....	....	0.00	....		....	....
Average.......	0.018	....	....	0.01	....	Not more than 0.02	0.3	50
(*e*) OXYGEN, O_2								
Content...............	....	....	....	....	0.004	....	....	....
(*f*) PHOSPHINE, PH_3								
Number of determinations	1	....	....	1	1	....	....	....
Content...............	0.00001	....	....	0.0001	0.0001	....	....	....
(*g*) AMMONIA, NH_3								
Content: Computed.....	0.00010	0.00025	0.00043	0.00062	....	0.00025	0.030	0.68
Observed......	....	....	....	0.00088	....	....	....	....
Observed......	....	....	....	0.00093	....	....	....	....
(*h*) METHANE, ETC. (BY DIFFERENCE)								
Content...............	65.5	68.7	69.7	70.3	....	67.6	7 760	182 400

matter, is converted to ferrous sulfide, indicates that the iron is in the form of a quite reactive compound. It seems quite probable that the sludge usually contains ferrous carbonate.

Equilibrium considerations lead to some interesting conclusions regarding the percentage of hydrogen sulfide in the gas from a sludge containing both ferrous carbonate, $Fe\,CO_3$, and ferrous sulfide, $Fe\,S$. The following equilibrium may be considered to exist:

$$\mathrm{Fe\,S + CO_2 + H_2O = H_2S + Fe\,CO_3} \qquad (4)$$

The equilibrium constant expression is:

$$\frac{p\,\mathrm{H_2S}}{p\,\mathrm{CO_2}} = K \qquad (5)$$

From data on the solubilities of ferrous carbonate [8] and of ferrous sulfide,[10] the value of K is computed to be 10^{-4}. This may be in error by as much as a factor of two due to uncertainty in the solubility of ferrous sulfide.

If the gas and sludge liquor were really in equilibrium with both ferrous sulfide and ferrous carbonate, and if the gas contains about 30% of carbon dioxide, the concentration of hydrogen sulfide should be about 0.003 per cent. Higher values might be expected if there is insufficient opportunity for reaction of the sulfides to gain equilibrium with the $Fe\,CO_3$. Low values might be found if a trace of air should get into the digestion tanks, because conditions would be favorable for a rapid oxidation of the hydrogen sulfide.

Referring to Table 15(*b*), it is seen that one sample of laboratory gas gave a value seventeen times as great as the computed value, and that, in other cases, the hydrogen sulfide was less than one-thirtieth of the computed value. Fortuitously, the average concentration of hydrogen sulfide is in fair agreement with the value of 0.003 per cent. The most that can be concluded is that, under the conditions of these observations, the equilibrium serves as a moderating factor, holding the percentage of hydrogen sulfide within limits.

According to theory, the concentration of hydrogen sulfide should be independent of the hydrogen-ion concentration as long as both iron compounds are present. Hence, it should be the same at all stages of digestion, except for small changes in the percentage of carbon dioxide. The data show no pronounced trend in the percentages of hydrogen sulfide by stages.

Analyses for mineral sulfides in the sludge show an increase of 153 lb per day in the digestion process. If all this sulfur formed hydrogen sulfide instead of being combined with iron, it would be equivalent to 0.7% of the gas. However, even if no iron were present, approximately one-half this hydrogen sulfide would remain in solution in the sludge liquor.

From the known solubility of hydrogen sulfide in water, it is calculated that sludge liquor in equilibrium with gas containing the average concentration of hydrogen sulfide (0.0043%) would contain about 0.13 ppm of free hydrogen sulfide. The total dissolved sulfides, calculated from the pH-value and the ionization constant [11] of hydrogen sulfide, should be about 0.3 ppm. The gas

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[10] *Zeitschrift für Anorganische Chemie*, Vol. 65, p. 136 (1909).

[11] "Relationship Between Free Hydrogen Sulfide and Total Sulfides in Sewage," by Richard Pomeroy, *California Sewage Works Journal*, Vol. VI, No. 2 (1934), p. 54.

(which, in one case, was found to contain 0.058% of hydrogen sulfide) must have come from liquor containing about 4 ppm of sulfides.

Carbon Monoxide, Hydrogen, and Phosphine.—Carbon monoxide was determined by the iodine pentoxide method, and hydrogen by combustion with palladium catalyst and weighing of the water. These methods do not positively establish the presence of either gas; they do establish upper limits for the concentrations. Phosphine was determined by oxidation with bromine water and subsequent determination of the phosphoric acid. The presence of phosphine in sludge gas has been suggested by other writers, but apparently has not previously been determined. The quantity is a mere trace.

Ammonia.—From the ionization constant of ammonium hydroxide, the concentration of ammonium ion in the liquor, and the pH-value, it is possible to compute the concentration of ammonium hydroxide, NH_4OH, in the liquor. From the known solubility of ammonia one may compute the equilibrium percentage of NH_3 in the gas. Table 16 shows the calculated NH_3 content of the gas in equilibrium with the various liquors which were analyzed.

TABLE 16.—COMPUTED VALUES OF AMMONIUM CONTENT

Liquor discharge	Ammonium nitrogen, in parts per million of nitrogen	Hydrogen ion concentration, pH	Ammonium hydroxide, NH_4OH, in parts per million of nitrogen	Percentage of ammonium, NH_3, in the gas
First tank	444	6.74	1.33	0.00016
Second tank	583	6.93	2.74	0.00033
Third tank	692	7.06	4.43	0.00053
Fourth tank	757	7.14	5.90	0.00071
Twenty-three days additional digestion	1 026	7.26	10.26	0.00123

The analyses of sludge liquors were made in all cases on the liquor flowing from one tank to the next. Hence, they do not represent the average composition of the liquors in the tanks. The gas from any tank would be expected to be intermediate in composition between the gases in equilibrium with the liquors entering and leaving that tank. It is upon this basis that values shown in Table 15 were estimated.

Ammonia in gas from the fourth stage was determined by bubbling a large volume through acid solutions and subsequently adding potassium hydroxide, KOH, and distilling and titrating. Two determinations gave 0.00088% and 0.00093% as against the computed value of 0.00062 per cent.

Olefines.—A volume of 1 800 liters of gas from the first stage was passed at a rate of about 20 liters per hr through bromine water and then into a 2-liter flask cooled with ice. At the end, the excess bromine was allowed to evaporate and the reaction tube and receiving flask were washed with ether. Evaporation of the ether solution left 2 or 3 grams of strongly lachrymatory solid and liquid bromine compounds.

If the gas contained ethylene, ethylene dibromide should be present in the product. An attempt was made to remove ethylene dibromide by vacuum distillation, but none could be obtained. If it was present, it certainly did not exceed 0.01%, and probably was not more than 0.001 per cent. From the solid portion of the mixture a small quantity of a compound was crystallized, which was identified as 1, 2, 3, 4 tetrabromo-butane, showing the presence of buta-diene ($CH_2 : CH \cdot CH : CH_2$) in the gas. However, the proportion of this olefine in the gas did not exceed 0.1% and possibly was not more than 0.001 per cent.

Other Hydrocarbons.—A sample of gas was analyzed by the research laboratory of a local oil company to determine the possible presence of hydrocarbons other than methane. Results were reported as follows:

Ethane, C_2H_6	0.04%
Propane, C_3H_8, and butane, C_4H_{10}	0.01%
Pentane, C_5H_{12}, and heavier	0.07%

Nitrogen.—The usual method of combustion in a combustion pipette is not satisfactory for determining quantities of nitrogen of the order of 2%, or less. No apparatus was available for more suitable methods.

Facts previously cited lead to the belief that very little nitrogen is present in the gas. Analyses of sludges to and from the digestion system did not indicate any loss of nitrogen from the sludge. Laboratory experiments indicated that the loss, if any, probably did not exceed 17 ppm. If this loss went to elemental nitrogen, it would amount to 0.12% by volume of the gas. Atmospheric nitrogen dissolved in the sewage (not included in the nitrogen analyses) would be gassed out, and under the conditions prevailing at this installation would yield about 0.09% of nitrogen in the gas. No other sources of nitrogen in the gas are apparent, except atmospheric contamination, and ordinarily such contamination is effectively excluded in this installation.

Methane.—Samples of the gas were passed through a train of caustic potash absorbers to remove the carbon dioxide, and the density of the residual gas was determined. The density of pure methane relative to air is 0.5544. Eleven determinations of the density of the methane fraction gave values from 0.562 to 0.577, with no consistent differences between the different stages. The average is 0.5678, 2.4% greater than the density of pure methane. The higher hydrocarbons known to be present should raise the density of the methane fraction about 0.6 per cent. The remainder of the effect is doubtless due to various other unindentified constituents.

SUMMARY AND CONCLUSION

(1) Results of a 4-yr study of the operation of a four-stage digestion system disclose that, with a nominal detention time of about 12 days, a reduction of 47% of the organic matter added was achieved. The sludge produced dried satisfactorily without odor or fly nuisance, and showed no appreciable difference in drying characteristics when compared with sludge digested three times as long.

(2) The results are also quite conclusive in determining that agitation by mechanical equipment is not needed in separate sludge digestion.

(3) Controlled seeding of raw solids with digested sludge, as accomplished in the multi-stage process, may be a factor in efficient digestion.

(4) The design of the tanks permitted a large accumulation of scum without reducing the efficiency of the system.

(5) The survey reveals that no appreciable liberation or absorption of heat occurred during the digestion process.

(6) The presence of relatively large quantities of iron salts in the sludge appears to have no deleterious effect upon the digestion process, and is beneficial in holding the concentration of hydrogen sulfide in the gas to a very low value.

(7) No nitrogen is lost in the digestion process. A portion is converted into soluble form and appears in the sludge liquor. The quantity remaining in the solids is approximately proportional to the organic matter remaining.

(8) The gas from the first stage is not substantially inferior to gas from the other stages as regards utilization for fuel; 47% of all gas was produced in the first stage.

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PAPERS

LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION

By Y. L. Chang,[1] Esq.

Synopsis

This paper describes a laboratory investigation of the problems of flume traction and transportation. Authoritative literature on the subject is also reviewed. Although the experiments are not—and cannot be expected to be—of the high accuracy obtainable in some classes of laboratory work, it is hoped that the results, when suitably corrected and refined, will provide a close guide to a scientific conclusion.

The subject is presented under three main headings: Part I is concerned principally with the tractive force required to cause initial movement of débris; the laboratory investigation by the writer is described in detail, an equation for critical tractive force is presented, and the results obtained by other investigators are tabulated and shown to conform generally to the same equation. Part II deals with the laws of transportation by traction, and Part III, with the laws of transportation by suspension. In each case the writer's experiments are reviewed briefly, while the major part of the presentation consists of theoretical analyses and discussions of the work of other experimenters. Formulas are suggested for making rough estimates of the traction load and the suspended load in either natural streams or regularly shaped channels. The nomenclature is summarized in the Appendix.

This paper is an abstract of a thesis[2] entitled "Hydraulics of Flume Traction and Transportation".

PART I.—FLUME TRACTION

The first part of the investigation was directed to determining the critical tractive force for a series of uniform sands of various sizes, and to studying the relation between the variables involved.

Note.—Discussion on this paper will be closed in **March, 1938**, *Proceedings*.

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[2] The thesis has been submitted as a partial fulfillment for the degree of Doctor of Philosophy to the Victoria Univ., Manchester, England. Original copies are also filed with the Board of Trustees, Nanking, China, and in the Library of the National Tsing Hua Univ., Peiping, China.

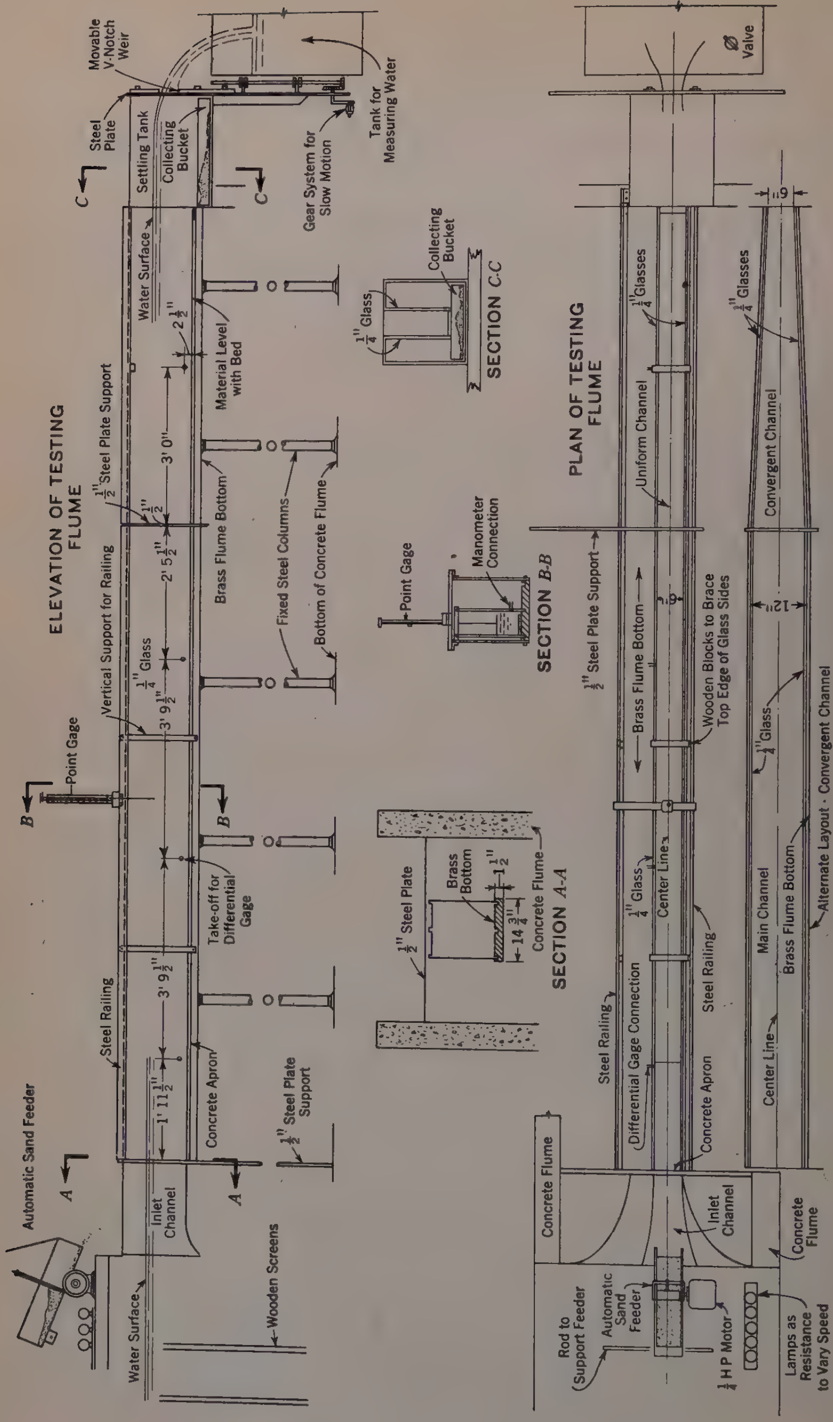

FIG. 1.—GENERAL ARRANGEMENT OF TESTING APPARATUS

Description of Apparatus

The laboratory flume and appurtenances are shown in Fig. 1. Three different arrangements were possible: (1) A uniform channel, 18 ft long and 12 in. wide; (2) a uniform channel, 18 ft long and 6 in. wide; and (3) a uniform channel, 12 ft long and 12 in. wide, followed by a 6-ft length of channel converging to a width of 6 in. at its down-stream end. The convergent section was provided in order to locate more exactly the point (and, hence, the velocity) at which movement commenced. The point-gage, Pitot tube, and templet for smoothing the bed were arranged to slide on rails on top of the flume.

The point-gage could be read to 0.005 in. The drop in water-surface elevation from point to point along the flume was measured accurately by means of a toluene differential gage, inclined at an angle of α from the horizontal. This arrangement of the apparatus produced a magnification of 46 to 1 when $\alpha = 10°$, and readings were taken with a traveling vernier microscope (Fig. 2) reading to 0.01 mm. Proper corrections were made for temperature, which was observed on a mercury thermometer in contact with the gage.

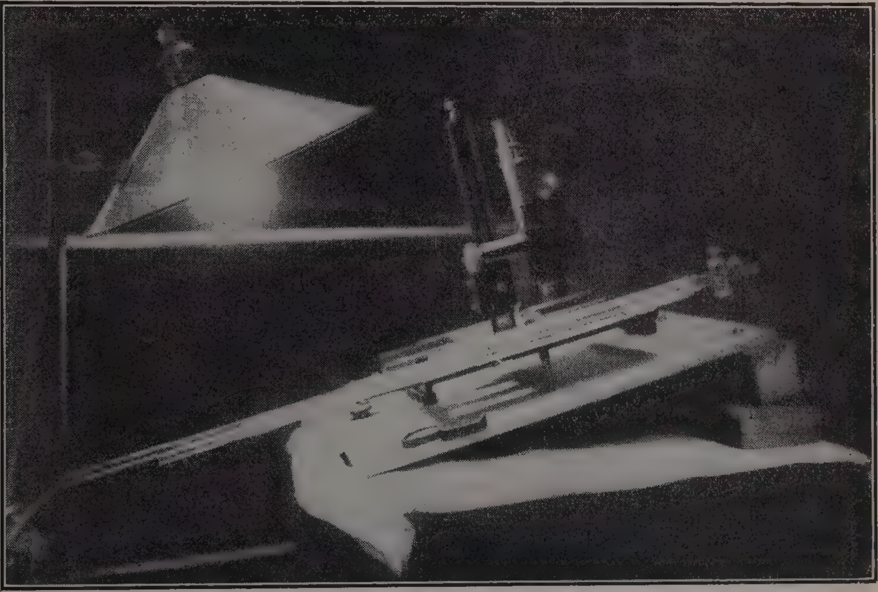

Fig. 2.—Toluene Differential Gage (Inclined) with Traveling Vernier Microscope

The water discharge could be measured either by means of a diaphragm in the feed-pipe or, volumetrically, in the tank at the down-stream end of the flume. Short match sticks, weighted to float vertically, were used for observations of surface velocity, and a Pitot tube connected to a toluene gage provided accurate readings of sub-surface velocities at any desired point.

Procedure

A special feature of the present experiment is that the bed of the flume was horizontal, instead of inclined to the same slope as the water surface.

The latter arrangement is theoretically sound, by the reasoning of uniform flow and parallel tractive force, but the necessity for it is doubtful. In model experiments the slope should never be sufficient to produce shooting flow; the permissible limit, for depths commonly used, is about 0.002. The actual velocity required to initiate movement of the samples used in the present experiments was much smaller than that corresponding to the limiting slope, the average value being about one-fourth of it. To mold the bed exactly to such a slope is very difficult, even with accurate apparatus, and, for smaller slopes, it is entirely impracticable.

With the bed horizontal, it would be possible to correct to the condition of uniform flow by subtracting the loss of potential head due to acceleration of flow, from the observed drop of surface level. However, in the present experiments, with a smooth unriffled bed, this loss never exceeded 0.0002 ft, and, being smaller than the deviation of the static readings from the differential gage, was neglected. Over a riffled bed, any such correction is meaningless, for the surface waves are of the same order of amplitude as the riffles.

In beginning a test run, the bed material was placed in the flume, wetted thoroughly, and made perfectly level and smooth by means of the brass templet and a trowel. Water was then introduced very gradually so as not to disturb the smooth bed (even tiny surface waves may produce riffles especially in shallow depths). After the flume had filled sufficiently, the test run was begun, the depth of flow being controlled by the movable weir plate at the end of the settling tank, and by the valve in the feed-pipe.

When sand movement began, the water discharge was kept constant for a while to make sure that the movement was due to the drag of the flow rather than to local irregularities. Then the flow was increased a little to generate stronger movement of the bed. This procedure was continued until data for "general movement" were obtained. (The term "general movement" applies to that condition in which grains of mean diameter are in motion so frequently that the numbers are not countable. Such motion should be general, not local. It is not strong enough to affect bed configuration and does not result in transportation of an appreciable quantity of material.)

In the uniform channel the slope of the water surface can be measured directly by the differential gage; in the convergent section the slope varies at different points, and can be determined more accurately by calculation, taking into account the different roughnesses on the bed and the sides.

MECHANICAL PROPERTIES OF TEST SAMPLES

In all, twelve commercially sieved samples were tested in the flume, each sample consisting of grains of more or less uniform size. The average grain diameter varied from 8.090 mm for the coarsest sample to 0.134 mm for the finest. The principal mechanical properties are listed in Table 1.

The method of determining the grain diameters requires some explanation. For each sample, 150 particles were selected arbitrarily and examined under a microscope. The two diameters, D_1 and D_2, are those lying in the plane parallel to the horizontal glass, and were measured during this examination. For the coarsest materials, D_1 and D_2 were measured by projecting the images of the grains on to a screen. The diameter, D_3, is the vertical dimension of the grain. In general, the shortest of the three, D_3, cannot be measured under the microscope, and must be computed from the grain volume, as measured by the displacement of a definite number of grains in water. For this computation, the general shape of the grains must be assumed. Each sample appeared to be composed of grains of two general shapes, which may be designated as "spherical" and "cubical," respectively. An examination of about 50 grains by eye permitted an approximate determination of the relative number of grains of the two shapes. The demarcation is not definite, and the estimate in Column (9), Table 1, is rough, but it does permit the computation with reasonable accuracy of D_3, as well as of the mean diameter, D, and the ratio, o, of the longest to the shortest diameter. For fine grains the volume of which was not measurable, D was assumed equal to the mean of D_1 and D_2, and o was taken as $\frac{D_1}{D_2}$.

TABLE 1.—MECHANICAL PROPERTIES OF TEST SAMPLES

Sample designation (1)	Specific gravity* (2)	Average volume of 1 grain, in cubic centimeters (3)	GRAIN DIAMETERS, IN MILLIMETERS				o (ratio of longest to shortest diameter) (8)	Approximate relative number of spherical and cubical grains (9)
			D_1 (4)	D_2 (5)	D_3 (6)	D (7)		
6 205	2.547	0.2375	12.100	8.400	3.760	8.090	3.21	2 : 1
6 204	2.558	0.1195	9.880	6.960	2.790	6.540	3.53	2 : 1
6 203	2.567	0.0619	8.150	5.490	2.240	5.290	3.65	2 : 1
6 202	2.590	0.02195	5.790	3.990	1.523	3.770	3.74	2 : 1
6 201	2.633	0.00800	3.580	2.707	1.320	2.540	2.71	2 : 1
6 200	2.634	0.00264	2.465	1.789	1.016	1.760	2.43	3 : 1
6 199	2.640	0.00033	1.171	0.884	0.544	0.886	2.16	3 : 1
6 198	2.643	....	0.612	0.396	....	0.396	1.55	....
6 197	2.645	....	0.312	0.226	....	0.226	1.40	....
Pumice†	2.053	....	0.350	0.230	....	0.289	1.52	....
Pumice W‡	2.016	....	0.401	0.234	....	0.317	1.72	....
Emery	3.890	....	0.240	0.180	....	0.210	1.33	....
Sand α	2.520	....	0.155	0.113	....	0.134	1.37	....

* Determined by displacement of water in specific gravity bottle. Each value listed is a mean of three determinations.

† Composed of black and white particles.

‡ Separate analysis of the white particles of the pumice sample. About 95% of the pumice particles transported, and collected in the settling tank, and practically all the pumice particles in suspension, were white.

The uniformity modulus, M, has been proposed by Hans Kramer,[3] M. Am. Soc. C. E., as a convenient expression for the distribution of sizes of the particles in a sand mixture. The uniformity modulus of the samples used in this experiment was not determined, but a rough plotting of the microscopic readings of the grain sizes showed that all samples had nearly the same distribution. They are assumed, therefore, to have the same uniformity modulus.

EQUATION OF VARIED FLOW

As many observations were taken in the convergent channel, application of the principles of non-uniform flow[4] is necessary to evaluate slope and also to study the tractive force. The general expression for non-uniform flow is:

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[3] "Sand Mixtures and Sand Movement in Fluvial Models", by Hans Kramer, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 798.

[4] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., N. Y., 1934, p. 24.

$$S = S_0 - \frac{dy}{dx} = \frac{V^2}{C^2 R} - \frac{Q^2}{g B^2 y^3} \frac{dy}{dx}$$

the nomenclature being explained in the Appendix.

For a rectangular section, since $Q = V B y$,

$$\frac{dy}{dx} = \frac{S_0 - \frac{V^2}{C^2 R}}{1 - \frac{V^2}{g y}} \qquad (1)$$

For a horizontal bed, $S_0 = 0$; and reducing to Manning's form,

$$\frac{dy}{dx} = \frac{1}{C^2 R \left[\frac{1}{g y} - \frac{1}{V^2} \right]} = \frac{n^2 V^2}{2.21 R^{4/3} \left[\frac{V^2}{g y} - 1 \right]} \qquad (2)$$

in which C has been replaced by $\frac{1.486 R^{1/6}}{n}$. Further,

$$S = - \frac{dy}{dx} = \frac{n^2 V^2}{2.21 R^{4/3} \left[1 - \frac{V^2}{g y} \right]} \qquad (3)$$

Hence, for any value of V less than $\sqrt{g y}$, the slope of the water surface in the convergent channel can be calculated by the ordinary Manning formula for uniform flow, and divided by the term, $\left[1 - \frac{V^2}{g y} \right]$, to correct for the non-uniform flow condition.

MANNING'S *n* FOR THE SAMPLES

The value of n for the flume as a whole is a function of n_g (the roughness coefficient of the glass sides) and n_s (the coefficient for the sample composing the bed). It was desired to determine separately the value of n_s.

The Manning formula, with R replaced by $\frac{A}{p}$, may be written,

$$V = \frac{1.486 A^{2/3} S^{1/2}}{(n^{3/2} p)^{2/3}}$$

In the present case, the denominator of the fraction may be equated as follows:

$$n^{3/2} p = n_s^{3/2} B + n_g^{3/2} d \qquad (4)$$

in which d is the depth of flow. Then, if n_g and n are known, n_s can be computed.

There are two methods of determining n_g: (1) Experimentally: Assuming the brass bottom to have the same roughness as the glass, observations on flow through the flume with the sand bed removed will yield the desired value; and (2) by calculation from a derived formula.

J. Allen[5] has derived a formula for the roughness coefficient in terms of the Reynolds number, which reduces to,

$$n = \frac{R^{1/6}}{35.3 \left(\frac{R\,V}{\nu}\right)^{3/40}} \quad \text{.......................(5)}$$

The relation holds provided the bed and sides of the channel are smooth.

Results of Methods (1) and (2) proved to be in reasonable agreement, and gave a mean value for n_g of 0.0103.

In the present experiments, there are two cases of uniform flow and one of non-uniform flow. The corresponding equations for n_s are:

1.—For uniform channel, 1 ft wide:

$$n_s = [(1 + 2\,d)\,n^{3/2} - 2\,d n_g^{3/2}]^{2/3} = [(1 + 2\,d)\,n^{3/2} - 0.00208\,d]^{2/3}$$

2.—For uniform channel, 0.5 ft wide:

$$n_s = [(1 + 4\,d)\,n^{3/2} - 4\,d n_g^{3/2}]^{2/3} = [(1 + 4\,d)\,n^{3/2} - 0.00405\,d]^{2/3}$$

3.—For the convergent channel (any width):

$$n_s = \left[\frac{(B + 2\,d)\,n^{3/2} - 0.00208\,d}{B}\right]^{2/3}$$

This expression is identical to one derived independently by R. E. Horton,[6] M. Am. Soc. C. E. He extended the method to a trapezoidal canal section, for which he gives the equation:

$$n = \left[\frac{n_b^{3/2} + 2\,z\,n_l^{3/2}}{1 + 2\,z}\right]^{2/3} \quad \text{.....................(6)}$$

in which n_b and n_l are the separate values of n for the bottom and the sloping sides, respectively, and z is the ratio of the length of one of the sloping sides to the width of the bottom. This formula is particularly applicable to natural canals with different roughness of bed and sides.

Values of n_s for the twelve test samples are given, with other pertinent data, in Table 2.

RELATION BETWEEN MANNING'S *n* AND SIZE OF PARTICLES

Manning's equation is dimensionally sound only if the friction does not depend upon viscosity (that is, only if the turbulence is fully developed) and n has the dimension of $(\text{length})^{1/6}$. It will be shown later that the Chezy coefficient is equal to $\left(\frac{g}{\kappa}\right)^{1/2}$, in which g is the acceleration of gravity and κ, the coefficient of skin friction, a dimensionless factor. Hence, C is of the dimension, $\frac{(\text{length})^{1/2}}{(\text{time})}$. Expressing Manning's n in terms of C, $n = \frac{1.486}{C} R^{1/6}$. Since n is independent of time, the constant, 1.486, must be of the same dimensions as C; hence, n is of the dimension, $(\text{length})^{1/6}$. The experimental Equation (5) gives the same result.

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[5] "Streamline and Turbulent Flow in Open Channels", by J. Allen, *Philosophical Magazine*, Vol. 17, 1934.

[6] "Separate Roughness Coefficients for Channel Bottom and Sides", by R. E. Horton, *Engineering News-Record*, Vol. 111, 1933, pp. 652–653.

TABLE 2.—MANNING'S n FOR SAMPLES
(Experiments done in a uniform channel, 0.5 ft wide)

Sample designation	S	Depth of flow, d	R	V	n	n_s	n_s (mean)
6 205	0.00060 0.00125	1.070 1.046	0.341 0.332	1.110 1.585	0.0160 0.0161	0.0276 0.0258	0.0258
6 204	0.00092 0.00140	1.000 0.950	0.333 0.328	1.470 1.740	0.0148 0.0152	0.0222 0.0227	0.0225
6 203	0.00094 0.00109	1.041 1.125	0.338 0.346	1.505 1.696	0.0148 0.0144	0.0225 0.0218	0.0222
6 202	0.00121 0.00119	0.948 0.666	0.327 0.285	1.712 1.443	0.0144 0.0154	0.0210 0.0211	0.0210
6 201	0.00128 0.00253	0.871 0.294	0.193 0.134	1.460 1.320	0.0122 0.0148	0.0187 0.0194	0.0190
6 200	0.00047 0.00059	1.080 0.658	0.342 0.284	1.190 1.093	0.0132 0.0142	0.0186 0.0186	0.0186
6 199	0.00019 0.00066	0.875 0.492	0.318 0.248	0.746 1.130	0.0128 0.0136	0.0166 0.0166	0.0166
6 198*	0.000133 0.000166	0.960 0.736	0.329 0.298	0.700 0.715	0.0116 0.0119	0.0141 0.0141	0.0141*
6 197	0.00025 0.00026	0.817 0.837	0.189 0.191	0.711 0.730	0.0109 0.0109	0.0127 0.0129	0.0128
Pumice	0.000206 0.000081	0.282 0.518	0.132 0.167	0.478 0.352	0.0116 0.0115	0.0129 0.0138	0.0134
Emery	0.000417 0.000588	0.453 0.196	0.161 0.109	0.842 0.743	0.0107 0.0112	0.0120 0.0119	0.0120
Sand, α	0.000089 0.000154	0.438 0.357	0.158 0.146	0.387 0.481	0.0106 0.0107	0.0113 0.0113	0.0114

* Experiment done in a uniform channel, 1 ft wide.

Strickler has proposed the equation,

$$n = 0.0150\, D^{1/6} \qquad (7)$$

in which D is the median diameter of the grains of the surface layer, in millimeters. Logarithmic plotting of the values of n for the samples shows exceedingly good coincidence to Equation (7). The equation of the straight line best fitting the plotted points (Fig. 3) is

$$n = 0.0166\, D^{1/6} \qquad (8)$$

D in this case being the mean diameter of the bed material, in millimeters. The difference in the value of the constant is probably due to the different geometrical shapes of the grains tested.

AVERAGE TRACTIVE FORCE

For conditions of uniform flow at depth, d, the tractive force per unit area opposed to the bed resistance is

$$T = w\, S\, d \qquad (9)$$

The expression[3] presupposes uniform flow and infinite width of channel. Referring to Fig. 4, the total tractive force in an irregular channel is

$$\Sigma T = T_1 + T_2 + T_3 + \cdots = w\,S\,(d_1\,p_1 + d_2\,p_2 + d_3\,p_3 + \cdots) = w\,S\,A$$

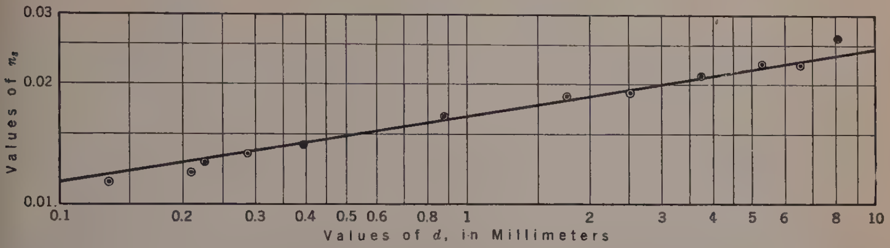

FIG. 3.—RELATION BETWEEN MANNING'S n AND MEDIAN DIAMETER OF SAND PARTICLES COMPOSING BED OF EXPERIMENTAL FLUME

and the average tractive force per unit area becomes,

$$\overline{T} = \frac{w\,S\,A}{p} = w\,S\,R \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The average unit tractive force in a rectangular flume should be

$$\overline{T} = w\,S\,d\,\frac{B}{B + 2\,d} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

According to Schoklitsch[7] an additional coefficient is required for channels in which the ratio of width to depth is less than 30. In such a case the average tractive force becomes, $\overline{T} = \frac{30}{32}\,w\,S\,d$, which is about 6% smaller than $w\,S\,d$. Schoklitsch[7] has suggested a similar, but more complete formula,

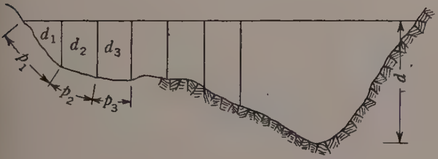

FIG. 4

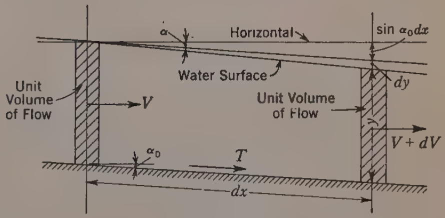

FIG. 5.—TRACTIVE FORCE IN ACCELERATING FLOW

$$T = \alpha\,w\,S\,d \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which $\alpha = \dfrac{1 + \dfrac{d}{B}\,i}{1 + \dfrac{2\,d\,n_l}{B\,n_b}\sqrt{1 + i^2}}$, in which n_b and n_l are, respectively, the Manning n for the bottom and the sides, and i is the horizontal over the vertical of the sides. When $n_b = n_l$ and $i = 0$, $\alpha = \frac{B}{B + 2\,d}$, which is identical to Equation (11).

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[7] "Wasserbauliche Strömungslehre", by P. Nemenyi, Leipzig, 1934; or, "Über Schleppkraft und Geschiebebewegung", by A. Schoklitsch.

EFFECT OF NON-UNIFORM FLOW ON TRACTIVE FORCE

Assume the flow to be non-uniform but accelerating (the flow in the flume with horizontal bed is of this kind) in a channel of infinite width (Fig. 5). The change in energy between two sections may be expressed as follows: Increase of kinetic energy + work done in overcoming bed resistance = decrease in potential energy.

For a column of water of unit volume:

$$w \frac{y}{g} \tfrac{1}{2}[(V + dV)^2 - V^2] + T\,dx = w\,y \frac{\sin\alpha + \sin\alpha_0}{2}\,dx \ldots\ldots (13)$$

in which T is the tractive force on the bed, assuming that all frictional loss is due to bed resistance. Neglecting higher orders of the derivative than the first and reducing, Equation (13) becomes,

$$T = \left[w\,y \frac{\sin\alpha + \sin\alpha_0}{2} - \frac{w\,y\,V}{g} \frac{dV}{dy} \frac{dy}{dx} \right] \ldots\ldots\ldots\ldots (14)$$

Taking $V = \frac{q}{y}$, in which q is the discharge per unit width of the channel, $\frac{dV}{dy} = -\frac{q}{y^2} = -\frac{V}{y}$. Also, $-\frac{dy}{dx} = \sin\alpha_0 - \sin\alpha$; and

$$T = \left[w\,y \frac{\sin\alpha + \sin\alpha_0}{2} + w \frac{V^2}{g} (\sin\alpha - \sin\alpha_0) \right] \ldots\ldots\ldots (15)$$

For a sloping bed with uniform flow, $\sin\alpha = \sin\alpha_0 = S$:

$$T = w\,S\,y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

For accelerating flow, $\alpha > \alpha_0$; for decelerating flow, $\alpha_0 > \alpha$. For accelerating flow on a horizontal bed, $\sin\alpha_0 = 0$:

$$T = w\;S \left(\frac{y}{2} + \frac{V^2}{g} \right) = \frac{w\,S}{2} \left(y + 4 \frac{V^2}{2\,g} \right) \ldots\ldots\ldots\ldots (17)$$

therefore, for high depth with comparatively small velocity, the tractive force on a horizontal bed is one-half that on a sloping bed with the same inclination of the water surface. In case $\sin\alpha_0 = -\sin\alpha$, that is, when the bed has an adverse surface slope, there is only a tractive force by the velocity head, namely, $w\,S \left(4 \frac{V^2}{2\,g} \right)$.

SUGGESTED EQUATION FOR CRITICAL TRACTIVE FORCE

The critical tractive force, T_0, has been defined as that tractive force which brings about general movement of the average grains of the bed material. This force, which is constant for a certain material, is not determined by either d alone or S alone, but by a constant product, $w\,S\,d$. Considering the tractive force as a function of the mechanical properties of the bed material, it can be written, $T_0 = F$ (grain size, grain distribution, grain form, apparent weight per unit volume in water) $= F\,(D^x\,M^{-1}\,n^\alpha\,\rho_e{}^y)$ in which Manning's n is assumed to be a function of the size and shape of the grains. It has been shown by Kramer[3] that tractive force is inversely proportional to M, which is a pure numerical ratio. Victor L. Streeter,[8] Jun. Am. Soc. C. E., has suggested that the coefficient of roughness varies directly with the depth and slightly with the interval of the grooves; for larger and more irregularly shaped grains, the grooves on the surface of the material would be expected to be deeper and larger. Therefore, it can be assumed that n, having a dimension of $(\text{length})^{1/6}$, takes the place of $(D\,o)^{1/6}$. By the method of dimensional analysis the following exponentials may be equated: $1 = y$; and $-1 = x - 2\,y + \frac{\alpha}{6}$. From Manning's formula, however, the tractive force, T, is proportional to n^2; therefore, $T_0 = c\,D^{2/3}\,\rho_e\,n^2\,M^{-1}$, in which c is a constant, and if n is assumed proportional to $(D\,o)^{1/6}$,

$$T_0 = \frac{c\,D\,\rho_e\,o^{1/3}}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

For uniform, equal sized grains of material, $M = 1$, and for spherical grains, $o = 1$, and the formula reduces to

$$T_0 = c\,\rho_e\,D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

which is dimensionally correct.

CRITICAL SIZE OF SAND FOR CRITICAL TRACTIVE FORCE RELATIONSHIP

Table 3 shows in detail the observations made on a typical sample to determine the critical tractive force, and Table 4 summarizes the results of tests on all samples. In Fig. 6, the mean values of T_0 are plotted logarithmically against the values of $\rho_e\,D\,o^{1/3}$, M being assumed to be unity for uniform material or a constant for uniform distribution. It will be noted that for sand of a larger size than No. 6198, a line inclined at 45° to the X-axis joins up the points smoothly; for sand less than that size the points are better fitted by a line inclined at approximately 26.5 degrees. This fact indicates that there is a critical value of the function, $\rho_e\,D\,o^{1/3}$, above which it is directly proportional to the critical tractive force, and below which it is proportional to the square of the latter quantity. It happens that these two lines intersect at the value of T_0 for $\rho_e\,D\,o^{1/3} = 1.0$. Expressed symbolically,

$$T_0 = 0.0175\left[\frac{\sigma - \rho}{\rho}\,D\,o^{1/3}\right]^\beta \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which $\beta = 1$ for $\frac{\sigma - \rho}{\rho}\,D\,o^{1/3}$ greater than 1, and $\frac{1}{2}$ for $\frac{\sigma - \rho}{\rho}\,D\,o^{1/3}$ less than 1.

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[8] "Frictional Resistance in Artificially Roughened Pipes", by Victor L. Streeter, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 681.

TABLE 3.—DETAILED TABULATION OF OBSERVATIONS ON A TYPICAL SAMPLE *

Run No. (1)	B, in feet (2)	d, in feet (3)	A, in square feet (4)	R, in feet (5)	Q, in cubic feet per second (6)	V, in feet per second (7)	$\frac{V^2}{2g}$, in feet (8)
1	0.496	0.728	0.3615	0.1852	0.224	0.620	0.006
2	0.495	0.609	0.3265	0.1800	0.214	0.654	0.007
3	0.495	0.591	0.2950	0.1760	0.212	0.753	0.009
4	0.496	0.454	0.2250	0.1603	0.134	0.596	0.006
5	0.496	0.282	0.1400	0.1320	0.081	0.575	0.005
6	0.496	0.242	0.1200	0.1224	0.071	0.591	0.005
7	0.496	0.143	0.0710	0.0905	0.048	0.680	0.007
8	0.496	0.125	0.0618	0.0830	0.041	0.663	0.007
9	0.495	0.461	0.2285	0.1613	0.131	0.572	0.005
10	0.495	0.301	0.1490	0.1357	0.081	0.546	0.005
11	0.495	0.250	0.1240	0.1245	0.084	0.677	0.007
12	0.496	0.175	0.0870	0.1027	0.049	0.566	0.005

TABLE 3.—(*Continued*)

Run No. (1)	R (9)	n (10)	S‡ (11)	$w\,S\,d$, in pounds per square foot¶ (12)	T,‖ in pounds per square foot¶ (13)	Movement (14)
1	8 410	0.01107	0.000204	(0.00933)	(0.00478)	General motion
2	8 650	0.01098	0.000230	(0.00956)	(0.00457)	General motion
3	9 775	0.01046	0.000286	(0.01070)	(0.00560)	General motion
4	7 100	0.01105	0.000227	(0.00642)	(0.00338)	General motion
5	5 690	0.01110	0.000276	0.00495	0.00256	Occasional motion
6	5 350	0.01094	0.000311	0.00480	0.00254	Occasional motion
7	4 675	0.01077	0.000600§	(0.00562)	(0.00320)	General motion
8	4 250	0.01136†	0.000677§	(0.00558)	(0.00323)	General motion
9	6 820	0.01056	0.000234§	(0.00681)	(0.00352)	General motion
10	5 640	0.01063	0.000220§	0.00420	0.00220	Occasional motion
11	6 440	0.01071	0.000384§	(0.00617)	(0.00333)	General motion
12	4 480	0.01136†	0.000371§	0.00417	0.00226	Occasional motion
....	....	....	....	0.0075	0.00395 $= T_0$	....

* Sand α; $\sigma = 2.520$; $D = 0.134$ mm.

† Assume $n = n_s$ for small depth.

‡ $S = \left[\frac{n\,V}{1.486\,R^{2/3}}\right]^2 \div \left[1 - \frac{V^2}{g\,d}\right]$.

§ Calculated from $n^{3/2} = \frac{0.00116\,B + 0.00208\,d}{B + 2\,d}$.

‖ $T = \frac{w\,S}{2}\left[d + 4\,\frac{V^2}{2\,g}\right]$ for channel of horizontal bed.

¶ The values in parentheses are taken to compute the mean value of the critical tractive force, T_0.

TABLE 4.—MEAN VALUES OF CRITICAL TRACTIVE FORCE FOR TEST SAMPLES
(M is assumed equal to unity)

Sample No.	$\frac{\sigma - \rho}{\rho}$	D, in millimeters	$o = \frac{D_e}{D_s}$	$o^{1/3}$	$\frac{\sigma - \rho}{\rho}\,D\,o^{1/3}$	$w\,S\,d$, in pounds per square foot	T_0, in pounds per square foot
6 205	1.547	8.090	3.21	1.475	18.450	0.3620	0.18500
6 204	1.558	6.540	3.53	1.523	15.520	0.2780	0.13900
6 203	1.567	5.290	3.65	1.540	12.770	0.2220	0.11100
6 202	1.590	3.770	3.74	1.552	9.300	0.1830	0.09100
6 201	1.633	2.540	2.71	1.395	5.790	0.0970	0.04850
6 200	1.634	1.760	2.43	1.345	3.870	0.0699	0.03490
6 199	1.640	0.886	2.16	1.293	1.880	0.0327	0.01660
6 198	1.643	0.396	1.55	1.157	0.753	0.0143	0.00757
6 197	1.645	0.226	1.40	1.119	0.415	0.0114	0.00635
Pumice	1.053	0.289	1.52	1.150	0.350	0.0098	0.00510
Emery	2.890	0.210	1.33	1.100	0.668	0.0142	0.00767
Sand α	1.520	0.134	1.37	1.111	0.226	0.0075	0.00395

For ordinary quartz sand, $\sigma = 2.65$; $\rho = 1$; and $o = 1.75$. Hence, the size of sand corresponding to the critical value of the function is approximately 0.5 mm. For spherical grains, $o = 1$; therefore, $D = 0.6$ mm (approximately). In other words, for ordinary quartz sand grains larger than 0.5 to 0.6 mm in mean diameter, the critical tractive force is proportional to $\rho_e\, D\, o^{1/3}$; for smaller sands it is proportional to the square root of that function.

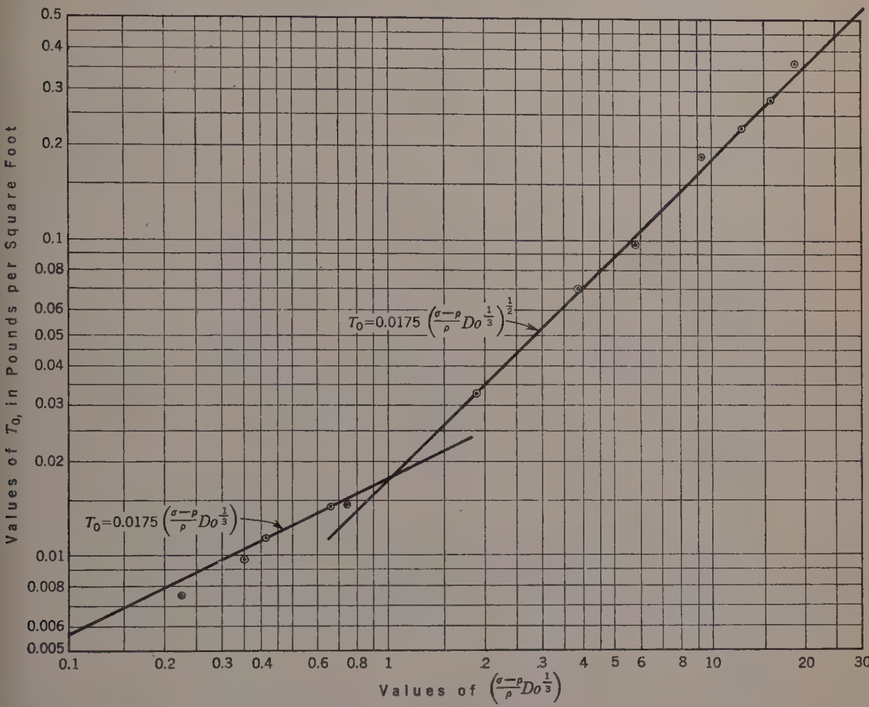

Fig. 6.—Critical Tractive Force (Chang Experiments; Horizontal Bed; M Assumed Unity)

Pertinent experimental data from all available sources are summarized in Table 5, and plotted in Fig. 7, with the uniformity modulus taken into account. (The values of o are not available and are considered unity.) To reduce to the condition of uniform flow, the value of T_0 of the writer's experiment should be reduced to one-half; but here, instead, the numerical values of $(\rho_e\, D\, o^{1/3})$ have been doubled to correspond to a uniform modulus of 0.5. Here, again, the results suggest the existence of a critical size of sand with respect to the critical tractive force relationship, although the demarcation of the two lines is, in this case, by no means distinct. A reason for this lack of definition will be advanced subsequently.

The two equations for T_0, represented by the lines on Fig. 7, are:

For $\left[\frac{\sigma-\rho}{\rho}\frac{D}{M}\right] > 2.0$:

$$T_0 = 0.00450\left[\frac{\sigma-\rho}{\rho}\frac{D}{M}\right]\ldots\ldots\ldots\ldots\ldots\ldots(21a)$$

TABLE 5.—DATA ON CRITICAL TRACTIVE FORCE DETERMINATIONS BY VARIOUS INVESTIGATORS

Item No. (1)	Investigator* (2)	Sample designation (3)	T_0, in pounds per square foot (4)	σ (5)	D, in millimeters (6)	M (7)	$\frac{\sigma-\rho}{\rho}\frac{D}{M}o^{1/3}$† (8)	$\frac{\sigma-\rho}{\rho}\frac{D}{M}$ for $M = \frac{1}{2}$ (9)
1	H. Kramer	I	0.01067	2.70	0.706	0.358	3.352	2.400
2		II	0.00800	2.70	0.558	0.461	2.055	1.898
3		III	0.01005	2.70	0.800	0.414	3.285	2.725
4	F. Schaffernak	*A*	0.01538	2.65	1.536	0.564	4.495	5.060
5	A. Schoklitsch	*B*	0.11160	2.60	6.520	1.000	10.430	20.860
6		*C*	0.05400	2.60	4.000	1.000	6.490	12.980
7		*D*	0.02318	2.60	2.256	1.000	3.618	7.236
8		*E*	0.00903	2.60	1.240	1.000	1.985	3.970
9		*F*	0.00574	2.60	0.916	1.000	1.467	2.934
10	H. Krey	*G*	0.00492	2.68	0.376	0.825	0.766	1.265
11		*H*	0.00574	2.61	0.526	0.869	0.976	1.695
12		*J*	0.00656	2.57	0.800	0.778	1.613	2.510
13	Prussian Experiment Institute	*K*	0.01046	2.65	1.154	0.265	7.200	3.810
14		*L*	0.00861	2.65	0.846	0.278	5.020	2.785
15		*M*	0.01190	2.65	0.836	0.366	3.780	2.765
16		*N*	0.01148	2.65	0.744	0.399	3.075	2.453
17		*O*	0.00492	2.65	0.244	0.435	0.926	0.805
18		*P*	0.01148	2.65	0.806	0.357	3.720	2.660
19		*Q*	0.01025	2.65	0.686	0.424	2.670	2.265
20	H. Engels	*R*	0.02050	2.65	1.484	0.229	10.650	4.880
21	G. K. Gilbert	*S*	0.00800	2.69	0.576	0.896	1.085	1.945
22		*T*	0.01251	2.69	1.906	0.714	4.507	6.340
23		*U*	0.06050	2.69	3.710	0.867	7.230	12.530
24		*V*	0.10050	2.69	5.296	0.820	10.900	17.860
25	E. Indri	*a*	0.00369	2.63	0.324	0.690	0.765	1.055
26		*b*	0.00636	2.70	0.474	0.481	1.673	1.610
27		*c*	0.04920	2.70	2.500	0.818	5.295	8.660
28		*d*	0.08370	2.70	4.000	0.777	8.750	13.600
29	Waterways Experiment Station	*e*	0.00970	2.65	0.586	0.280	3.460	1.936
30		*f*	0.00880	2.65	0.541	0.439	2.038	1.790
31		*g*	0.00880	2.65	0.525	0.539	1.607	1.733
32		*h*	0.00840	2.65	0.506	0.406	2.055	1.670
33		*i*	0.00800	2.65	0.483	0.438	1.820	1.595
34		*j*	0.00600	2.65	0.347	0.643	0.891	1.145
35		*k*	0.00660	2.65	0.310	0.525	0.975	1.023
36		*l*	0.00470	2.65	0.205	0.560	0.604	0.677
37		*m*	0.05800	2.65	4.077	0.566	11.870	13.470
38	Y. L. Chang	6205	0.18500	2.547	8.090	1.000	18.450	36.900
39		6204	0.13900	2.558	6.540	1.000	15.520	31.040
40		6203	0.11100	2.567	5.290	1.000	12.770	25.540
41		6202	0.09100	2.590	3.770	1.000	9.300	18.600
42		6201	0.04850	2.633	2.540	1.000	5.790	11.580
43		6200	0.03490	2.634	1.760	1.000	3.870	7.740
44		6199	0.01660	2.640	0.886	1.000	1.880	3.760
45		6198	0.00757	2.643	0.396	1.000	0.753	1.506
46		6197	0.00635	2.645	0.226	1.000	0.415	0.930
47		Pumice	0.00510	2.053	0.289	1.000	0.350	0.700
48		Emery	0.00767	3.890	0.210	1.000	0.668	1.336
49		Sand α	0.00395	2.520	0.134	1.000	0.226	0.452

---

* Data of Table 5 were taken from "Studies of River Bed Materials and Their Movement with Special Reference to the Lower Mississippi River, *Paper 17*, U. S. Waterways Experiment Station.

† o assumed unity for others than the writer's results.

and for $\left[\frac{\sigma - \rho}{\rho} \frac{D}{M}\right] < 2.0$:

$$T_0 = 0.00635 \left[\frac{\sigma - \rho}{\rho} \frac{D}{M}\right]^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

At the intersection, $\frac{\sigma - \rho}{\rho} \frac{D}{M} = 2.0$; hence, for $M = 0.5$, $D \sim 0.5$ mm. If the value of $M = \frac{1}{2}$ be substituted into the equations they become identical:

$$T_0 = 0.00900 \left[\frac{\sigma - \rho}{\rho} D\right]^{\beta} \ldots\ldots\ldots\ldots\ldots\ldots (21c)$$

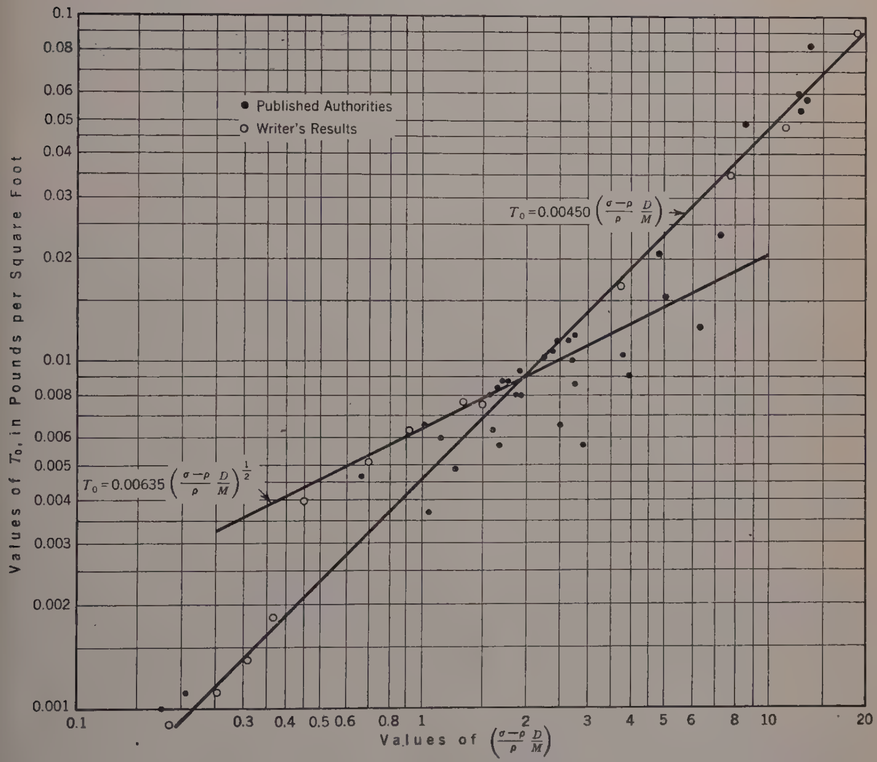

Fig. 7.—Critical Tractive Force (Data from Various Sources; o Assumed Unity; Plotting Adjusted to $M = \frac{1}{2}$)

The numerical coefficient in Equation (21c) is just about one-half that in Equation (20), showing that the tractive force needed to move débris on a sloping bed parallel to the water surface is only one-half that needed on a horizontal bed. It also indicates that the uniform modulus, M, has practically nothing to do with the critical tractive force, and could be omitted.

The equation suggested by Kramer for critical tractive force is,

$$T_0 = 0.00339 \left[\frac{\sigma - \rho}{\rho} \frac{D}{M}\right] \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Equation (22) has been modified by the United States Waterways Experiment Station[9] to

$$T_0 = 0.00595 \left[\frac{\sigma - \rho}{\rho} \frac{D}{M} \right]^{0.5} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

It is clear that both formulas are derived from the same data; then, if both are right, there must be a criterion separating the available data into two sets satisfied, respectively, by Equations (22) and (23). In this connection it is interesting to note the following quotation from the Station's report:[10]

> "There must be some limiting size of particle at which the larger size no longer moves first, and above which the finer materials are the first to be placed in motion. While no data were recorded in these experiments which would tend to locate this limiting size, it is believed that its value is in the vicinity of 0.6 mm."

In a paper by Morrough P. O'Brien,[11] Assoc. M. Am. Soc. C. E., the existence of such a criterion has also been outlined.

TURBULENCE AS AN INTERPRETATION

Dimensionally, the idea that the critical tractive force is proportional to $\rho_e\, D\, o^{1/3}$, taking o as a dimensionless ratio, is correct. In case it is proportional to $[\rho_e\, D\, o^{1/3}]^{1/2}$, the coefficient by which the function is multiplied will no longer be dimensionless. The turbulence of the flow is suggested as an explanation of this variation. The action of the tractive force on the bed material is equivalent to the motion of a fluid resisted by a tangential stress at the wall in a channel or in a pipe. It is reasonable to presume that this unit stress, T_0, will be a function of the depth, d (or hydraulic radius, R), density, ρ, viscosity, μ, and mean velocity, V; that is, $\tau_0 = C_1\, d^x\, \rho^y\, \mu^z\, V^k$. By dimensional analysis, the form of this function[12] is found to be

$$\tau_0 = C_1\, d^{k-2}\, \rho^{k-1}\, \mu^{2-k}\, V^k \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

For laminar flow, $k = 1$, and is constant because the flow is independent of the roughness of the wall. For turbulent flow, the value of k varies from 1.75 for extremely smooth channels to a maximum of 2 for channels which are rough. For $k = 1$, $\tau_0 = C_1 \frac{\mu}{d} V = K_1\, V$, and for $k = 2$, $\tau_0 = C_1\, \rho\, V^2 = K_2\, V^2$. These relations are sketched logarithmically in Fig. 8, and can be expressed in the form, $\tau_0 = K\, V^\alpha$, in which $\alpha = 1$ or 2.

Tractive force in turbulent flow varies with the square of the mean velocity, whereas tractive force in laminar flow varies directly with the mean velocity; consequently, if it is reasonable to presume that the tractive force in turbulent flow is a certain function of the sand, then the tractive force in laminar flow will be the square root of the same function, because in such flow the function of sand is equivalent to the reaction of the shearing stress due to velocity. Therefore, in the general equation,

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[9] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River", U. S. Waterways Experiment Station, *Paper 17*, 1935, pp. 29–30.

[10] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River", U. S. Waterways Experiment Station, *Paper 17*, 1935.

[11] "Notes on the Transportation of Silt by Streams", by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, 1936.

[12] "Modern Conceptions of the Mechanics of Fluid Turbulence", by Hunter Rouse, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., January, 1936, p. 21.

$$T_0 = C\left[\frac{\sigma - \rho}{\rho} D\, o^{1/3}\right]^{\beta} \qquad (25)$$

in which C is a constant, the experimentally determined values of $\frac{1}{2}$ and 1 for β appear to have a rational justification. Equation (25) is also plotted in Fig. 8. The discontinuity of the curve is here assumed to occur at the same ordinate as that of the curve showing the tangential stress relationship. With regard to the latter curve, there is a zone of uncertainty in the transition region between laminar and turbulent flow. If the relationships of tractive force with velocity and sand properties are justified, there must be also a zone of uncertainty in the curve of tractive force. The nature and the extent of this zone are not definite.

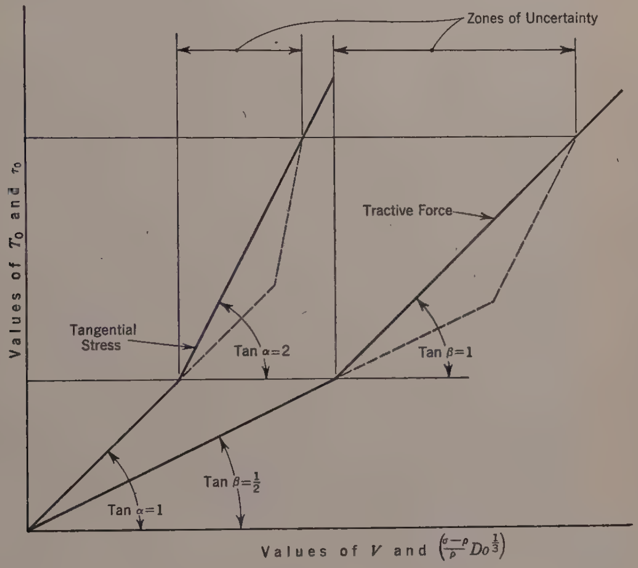
Zones of Uncertainty
Tractive Force
Values of T_0 and τ_0
Tangential Stress
Tan α=2
Tan β=1
Tan β=$\frac{1}{2}$
Tan α=1
Values of V and $\left(\frac{\sigma-\rho}{\rho} Do^{\frac{1}{3}}\right)$

FIG. 8

At the lower critical point, the shearing stresses in laminar and turbulent flow are identical,

$$\frac{\tau_s}{\tau_t} = \frac{\dfrac{2\mu}{R} V}{\dfrac{f}{4}\rho \dfrac{V^2}{2}} = \frac{16}{f}\,\frac{\mu}{V R \rho} = \frac{16}{f}\,\frac{1}{\mathsf{R}} = 1$$

Assuming 0.01 to be the average value of f, the roughness factor, in the experiment, R = 1 600; while Allen[5] gives 1 400 as the lower criterion. The upper criterion is uncertain; according to the writer's experiment it may be as much as ten times the value of the lower criterion. (As shown in Fig. 8, the zone of uncertainty on the curve for sand properties is considerably larger than that on the curve for velocity.) The same problem may be explained from another point of view by the theories of frictional drag given in hydro-mechanics.[13]

Summary: Part I

(1) In the convergent channel the water surface slope can be calculated by Equation (3).

(2) When the glass-sided flume used in the writer's experiments has bed material leveled on the bottom, Manning's n_s for the sand can be computed by,

$$n_s = \left[\frac{(B + 2\,d)\,n^{3/2} - 0.00208\,d}{B} \right]^{2/3}$$

assuming that Manning's n_g for the sides of the flume is 0.0103.

(3) The relationship between Manning's n and the mean diameter, D, in millimeters, of the grains is well expressed by Equation (8).

(4) For channels in which the ratio of width to depth is less than 30, Equation (12) may be used to find the tractive force on the bottom of either rectangular or trapezoidal sections.

(5) Water flowing at relatively low velocity in a deep channel with horizontal bed exerts a tractive force on the bottom equal to approximately one-half that exerted in uniform flow. In other words, the resistance to motion of particles on a horizontal bed is nearly double their resistance on a sloping bed having the same inclination as the water surface.

(6) A complete formula for the critical tractive force in terms of the mechanical properties of the sand is Equation (25).

(7) There is a critical size of sand at which the power index theoretically changes suddenly. For quartz sand the critical mean diameter of grain is about 0.5 mm. Turbulence of flow best explains this criterion.

PART II.—TRANSPORTATION BY TRACTION

Procedure

The experiments on transportation by traction were performed in the flume $\frac{1}{2}$ ft wide. Damp material was placed on the bed and leveled. Water was then admitted gradually, to the desired depth; and the flow was adjusted to a condition just above that causing general motion. The period for measuring sand discharge—that is, the time between inserting and withdrawing the collecting bucket—was chosen to be nearly that during which a constant rate of sand movement prevails. For small sand discharges a long period is desirable for the sake of accuracy. For greater discharges the period should be as short as possible, since the surface of the bed changes its form from instant to instant. The dry weight of the collected sand, divided by the measuring period, gives the rate of transportation, G.

The sand was fed automatically, but not accurately. A box, 18 in. by 6 in. by 6 in., filled with the sample material was supported on a rotating shaft connected directly to a $\frac{1}{4}$-hp motor the speed of which could be roughly controlled by a resistance composed of lamps. In the middle of the box there was a baffle-plate that could be moved vertically by screws. The rate of discharging the sand, therefore, could be approximately adjusted by the opening of the baffle-plate, through which the sand was sliding. The criterion to adjust the feeding, however, was based only upon the visual observation of the movement of the sand in the channel.

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[13] "Applied Hydro- and Aeromechanics", by L. Prandtl and O. G. Tietjens, N. Y., 1934, p. 86.

The depth measured was in all cases the depth to the original smooth bed surface. Slope measurements were made as in Part I. With the higher flows, however, more trouble was experienced in attaining a stable reading. As soon as riffles developed, the differences of readings on the differential gâge increased suddenly, and thereafter continued to increase throughout the run. This means that the roughness of the bed was increasing constantly from the time the riffles were formed, other factors being considered almost unaltered. However, the exact rate at which the slope varied is unknown. The mean value of several slope observations, taken desirably at equal time intervals, was assumed to represent the average condition during the run. The length of time of collection varied from 15 min to 1 hr, depending mainly on the intensity of movement. During each run, at least two measurements of water discharge were taken, most of them directly from measuring tank. Types of flow could be easily distinguished with light on the opposite side of the glass flume.

Observations on sand movement and riffle development were carefully noted at different stages. Riffles usually appeared first on the bed at the two ends of the channel, indicating disturbances in those regions, and at isolated points where there were irregularities of flow. Their formation could not be controlled.

DU BOYS' FORMULA

The formula originally derived by du Boys[14] in 1879 is of the form:

$$G = \psi\, T\, (T - T_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

in which T is the unit tractive force on the bed; T_0 is the critical tractive force required to initiate general movement; and ψ is a function of the size, etc., of the sand grains. Equation (26) can be written in many other forms by means of Manning's formula.[15] The derivation of Equation (26) was based upon the assumption that sand is moved in layers subject to uniform tractive force, and that the velocity gradient in the moving sand layers is linear. The coefficient of friction of sand in water is also considered constant. According to Fig. 9, the sand does not move at the layer subjected to the critical tractive force. The total tractive force consumed per unit area of sand is, $w\, S\, d = f\, (\gamma - w)\, n\, t$, in which f is the coefficient of friction of the sand in water; n, the number of layers of sand, which depends on the tractive

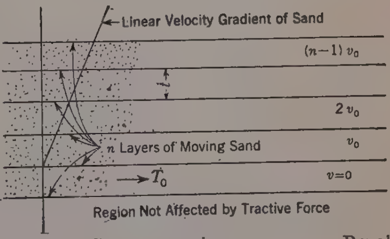
Linear Velocity Gradient of Sand
$(n-1)\, v_0$
t
$2\, v_0$
n Layers of Moving Sand
v_0
T_0
$v=0$
Region Not Affected by Tractive Force

FIG. 9.—CONDITIONS ASSUMED IN DU BOYS' EQUATION

force and the sand properties; and t, the thickness of each layer. The rate of sand transportation is, therefore,

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[14] "Le Rhone et les rivieres a lit affouillable", par P. du Boys, *Annales des Ponts et Chaussées*, Vol. 2, 1879.

[15] "Hydraulic and Sedimentary Characteristics of Rivers", by L. G. Straub, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1932.

$$G = t\,w\,[0 + v_0 + 2\,v_0 + 3\,v_0 + 4\,v_0 + \cdots (n-1)\,v_0] = n\,(n-1)\,\frac{w\,t\,v_0}{2}$$

No layer will be moved on which the force is less than the critical tractive force required to overcome the frictional resistance of the bed; therefore, $T_0 = f\,(\gamma - w)\,t$. Hence, $n\,T_0 = w\,S\,d$, and $n = \frac{w\,S\,d}{T_0} = \frac{T}{T_0}$, so that,

$$G = \frac{w\,t\,v_0}{2}\,\frac{T}{T_0}\left(\frac{T}{T_0} - 1\right) = \frac{w\,t\,v_0}{2\,T_0^2}\,T\,(T - T_0) = \psi\,T\,(T - T_0) \ldots . (27)$$

Using Chezy's equation, $V = C\sqrt{S\,d}$,

$$G = \psi\,w^2\,\frac{V^2}{C^2}\left(\frac{V^2}{C^2} - \frac{T_0}{w}\right) \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

Equation (28) indicates that transportation of sand will not occur until $V > C\sqrt{\frac{T_0}{w}}$; thereafter, the rate of transportation will be approximately proportional to the fourth power of the velocity.

The foregoing derivation applies only to the case in which materials are being dragged on a smooth bed; load due to saltation, which is closely associated with the formation of riffles, is not included. Although the derivation is not very sound, the resulting expression checks well with most experimental formulas, and is satisfactory for estimating sand movement in natural streams.

SAND MOVEMENT IN TERMS OF ADVANCE OF RIFFLES

The foregoing discussion applies only to a smooth bed. For a riffled bed, a better interpretation is not yet practicable.

In this paper, the velocity of sand movement is considered to be the same as that of the progression of the riffles. Of course, this assumption is only applicable in the velocity range in which the riffles are progressing in a downstream direction. The plotted results (Fig. 10) of the experiment, although the points are considerably scattered, could still indicate the following relation between the advancement of riffles and the velocity of the flow:

$$v_r = [0.000125 \text{ to } 0.001085]\,V^5 \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

both velocities being in feet per second. (The inclination of the lines is not absolute, because of the scattering of the plotted points, but they are approximately consistent to one another.) It is interesting to compare these results with those of Owens and Deacon.[16] The former gives $G \propto V^6$, whereas the latter indicates $G \propto V^4$. According to Gilbert's[17] observations on natural streams the power of V is much less than 6. His formula, based on flume tests, is, however, of the form $G \propto V^4$. Such deviation could probably be attributed to the different size of flume used.

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[16] "The Settlement and Transport of Sand in Water", by J. S. Owens, *Engineering*, 1912.

[17] "The Transportation of Débris by Running Water", by G. K. Gilbert, *Professional Paper 86*, U. S. Geological Survey, 1914.

It is observed that saltation is strongest when the riffles are the largest. Very roughly it can be set forth, that,

$$G \propto [(\text{height of riffle})^{\alpha} \times (\text{velocity of riffle})^{\beta}]$$

If it may be assumed that the size of a riffle, ξ, is equivalent to that of a grain,

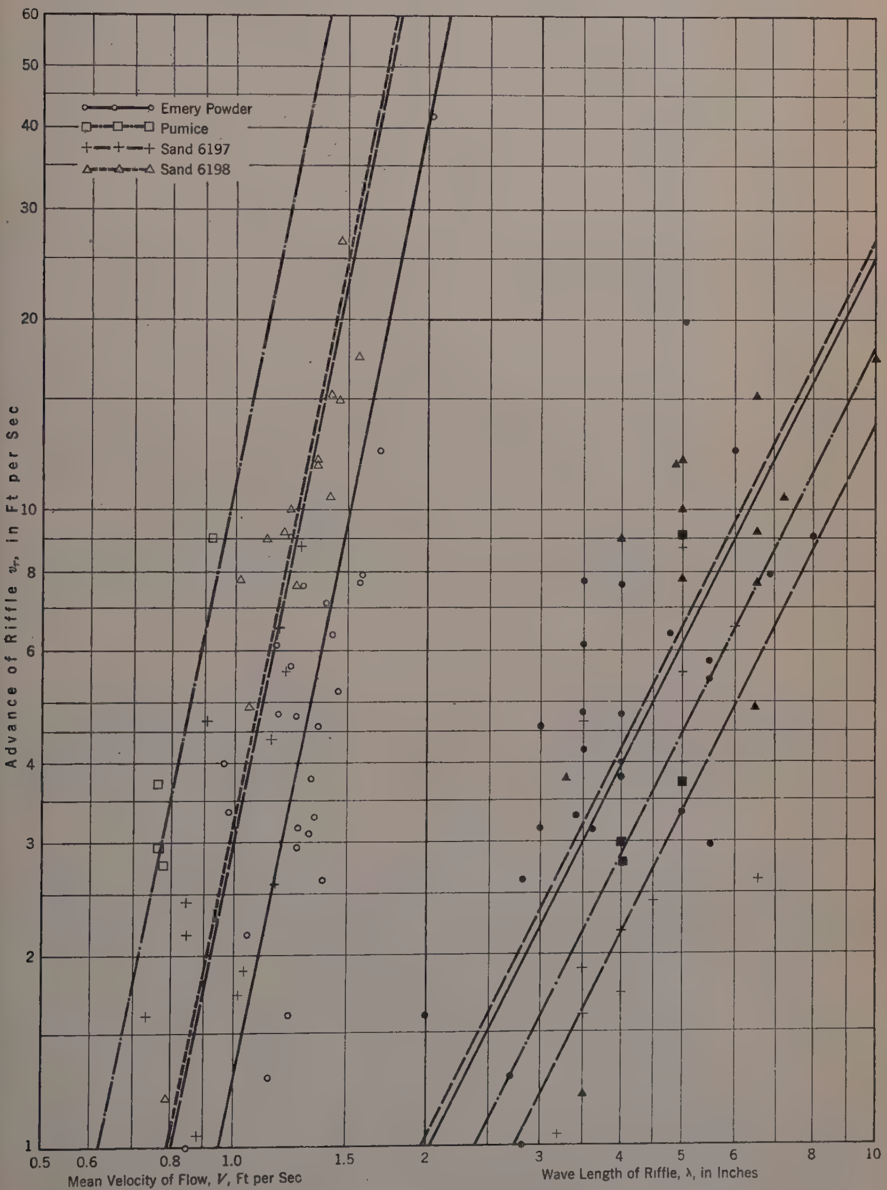

FIG. 10.—ADVANCE OF RIFFLES

and, consequently, that Manning's n is a direct measure of the size of the riffle, then the quantity of sand moved must be also a function of the size of the riffles. The United States Waterways Experiment Station found[10] also that the size of the riffles increased with the percentage of fine material in the mixture; but, obviously, the rate of transportation of a finer mixture is greater unless cohesion comes appreciably into effect. Observations[3] on the Rhine indicate that there is a general increase in the value of Manning's n with an increase in bed-load movement. It has been shown that Manning's n is dimensionally a function of the size of the particles to the one-sixth power. Actual plottings (as in Fig. 11) of the relation of G and n show that G is proportional to n^6, indicating that the exponent, α, should be unity. The deviation of points at higher values of n is due to the restraining effect of the narrow flume; that is, the narrowness limits of the development of larger riffles. When β is also unity, as assumed, the equation becomes:

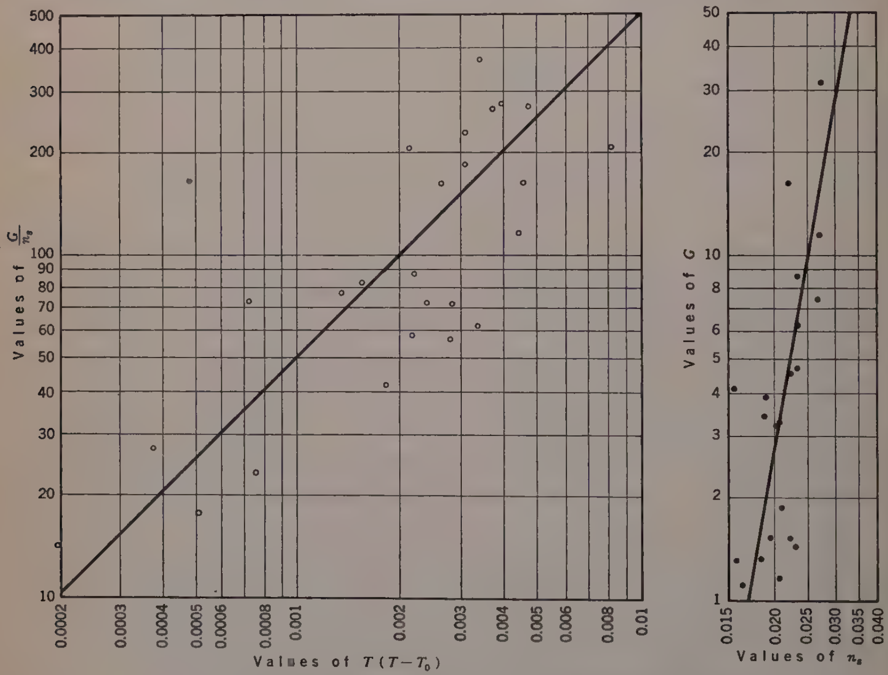

FIG. 11.—DATA ON TRANSPORTATION BY TRACTION: EMERY POWDER

$$G \propto \xi\, V^5 \qquad (30)$$

Using Chezy's formula for an infinite width of channel,

$$G \propto n^6 \frac{(S\, d)^{5/2}}{n^5} \propto n\, T^{2.5} \qquad (31)$$

The appearance of n in Equation (31) is justified dimensionally by noting that $T \propto n^2$.

EXPERIMENTAL FORMULA

A formula of the du Boys' type appears to fit best the data obtained in the present experiment, although the plotted points are so scattered that no equation will join them closely. When $\frac{G}{n}$ is plotted against $T(T - T_0)$ logarithmically, as in Fig. 11, lines with an inclination of 45° evidently satisfy the general requirements. The equations of these lines for three of the bed materials tested, read:

For emery,

$$G = 50\,000\, n\, T\,(T - T_0) \qquad (32a)$$

For 6197,

$$G = 100\,000\, n\, T\,(T - T_0) \qquad (32b)$$

For pumice,

$$G = 210\,000\, n\, T\,(T - T_0) \qquad (32c)$$

Since the numerical coefficient in Equations (32) must be the function, ψ, it can be written that

$$G = \frac{K\,n}{T_0^2}\,T\,(T - T_0) \qquad (33)$$

in which T_0 is defined by Equation (20) and K is a constant.

PUBLISHED AUTHORITIES

The following representative formulas given by other authorities are briefly reviewed herein as comparisons:

I.—A Schoklitsch.—Using additional experimental data of his own, but mainly the classic flume measurements by Gilbert, Schoklitsch developed the following formula[18] for uniform quartz grains of approximately the same density:

$$G = \frac{436.5}{\sqrt{D}}\,S^{1.5}\,B\,(q - q_0) \qquad (34)$$

in which G is the bed-load movement in pounds per second, D is the grain diameter, in millimeters, and q_0 is the critical discharge to move sand per unit width of channel, in cubic feet per second. The value of the latter is

$$q_0 = 0.00021\,\frac{d}{S^{4/3}}$$

II.—M. Fabre.—The results[19] obtained by the laboratory at Zurich, Switzerland, on gravels ranging between charcoal, the density of which approximates that of water, and baryta (protoxide of barium), of specific gravity greater than 4.0, lead to the following general relationship:

$$G = 2.16\,\frac{S^{1.5}\,Q}{\rho_e^{1/3}} - 20.70\,D\,\rho_e^{7/9} \qquad (35)$$

The equivalent critical discharge is $Q_0 = k\,\frac{\rho_e^{7/9}\,D}{S^{3/2}}$

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[18] "The Schoklitsch Bed Load Formula," by S. Shulits, Assoc. N. Am. Soc. C. E., *Engineering*, June, 1936.

[19] "Mesure des debits solides des cours d'eau", par M. Fabre, *Annales des Ponts et Chaussées*, Vol. 8, 1934.

III.—Meyer-Peter.—In studying the results obtained with gravels of the same density but of different diameters, and with different slopes, the Swiss engineers[20] have arrived at a relationship of the form:

$$G = a\, S^{3/2}\, Q - b\, D^{3/2} \quad \text{(36)}$$

and the equivalent critical discharge is

$$Q_0 = k \left(\frac{D}{S} \right)^{3/2}$$

in which a, b, and k are constants.

IV.—G. K. Gilbert.—The final expression by Gilbert[17] for bed-load movement, in which the slope, discharge, and fineness (reciprocal of D) are considered as of primary importance, is

$$G = k\, F\, (S - S_0)^{0.93 \text{ to } 2.37}\, (Q - Q_0)^{0.81 \text{ to } 1.24} \left(\frac{1}{D} - \frac{1}{D_0} \right)^{0.50 \text{ to } 0.62} \quad \text{(37)}$$

or, approximately,

$$G = \frac{a}{D^{0.58}}\, S^{1.59}\, Q^{1.02} - b \quad \text{(38)}$$

in which F is a shape factor, which is constant for a given ratio of depth to width of channel, and k, a, and b are constants. Those notations with the subscript, 0, are values for competent conditions to cause transportation.

V.—United States Waterways Experiment Station.—Data on the rate of sand movement obtained from experiments at the U. S. Waterways Experiment Station[10] conformed to the following relationship (after the formation of riffles):

$$G = \frac{k}{n}\, (T - T_0)^{1.5 \text{ to } 1.8} \quad \text{(39)}$$

The contradiction of Equation (39) with that of the writer seems that of the reciprocal arrangement of Manning's n. In *Paper No. 17*, of the Experiment Station,[10] it has been concluded that "in general, the finer the sand mixture, the larger was the riffle size, and, consequently, the higher the value of n". Now, from all formulas given herein, G is always inversely proportional to a certain function of D; therefore, the sand movement is greater for finer material. If the finer material is more favorable for higher riffle development, then the sand movement must vary directly instead of inversely with Manning's n if a first power is used.

Exner's[21] expression for dune (riffle) velocity is

$$v_r = \frac{k}{(d - \xi)^2} \quad \text{(40)}$$

in which ξ is the height of the surface of the dune, in feet. This, again, indicates that the larger the dune, the faster it will travel.

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[20] "Neure Versuchsresultate über den Geschiebetrieb", von E. Meyer-Peter, H. Favre, und A. Einstein, *Schweizeritche Bauzeitung*, No. 103, 1934, *Professional Paper 86*.

[21] "Uber die Wechselwirkung zwischen Wasser und Geschiebe in Flussen", von F. M. Exner, Wien, Aka. Sci. Ber., Vol. 134, 1935.

VI.—C. H. MacDougall.—The form of equation that has been found to fit the data accumulated at the Massachusetts Institute of Technology[22] is

$$G = a\,S^{b}\,(Q - Q_0)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(41)$$

The constants, a and b, are interrelated, and also dependent upon D and M. They range from 100 to 1 000 and from 1.25 to 2.00, respectively.

VII.—H. Nakayama.—With certain restricted conditions, possible variations of bottom slopes, discharges, modes of flow, etc., were experimented with, and the results[23] obtained yielded the relation:

$$G = (\alpha + \beta\,S)\,Q\,(V^2 - V_0^{2})$$

in which α is found to be small compared with $\beta\,S$. Further, assuming that V_0^{2} is small compared with V^2, and may be neglected,

$$G = \beta\,S\,B\,d\left[\frac{1.486}{n}\right]^3 (S\,d)^{3/2} = \frac{K}{n^3}\,T^{2.5}$$

Manning's n may be taken as proportional to $D^{1/6}$ or $\xi^{1/6}$, so that, $G = \frac{K}{\sqrt{D}}\,T\,(T - T_0)$, or,

$$G = \frac{K}{\sqrt{\xi}}\,T\,(T - T_0)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(42)$$

VIII.—S. D. Chyn.—From experiments made at the Massachusetts Institute of Technology on sands bigger than No. 48 mesh ($d > 0.5$ mm), S. D. Chyn[24] has obtained the following formula:

$$G = \psi\,V^{p}\,(T - T_0)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(43)$$

in which ψ is a function of the sand. The exponent, p, being a function of Manning's n, has three distinct values for any one material, corresponding, respectively, to a smooth bed, to a bed being dragged, and to a riffled bed. That is, before riffles have developed, p is a function of the diameter of the sand grains; after their formation, it is a function of the size of the riffles.

IX.—F. Schaffernak.—Materials used in Schaffernak's[25] tests were obtained from the Danube River; experiments were performed both on samples of uniform size and on definite mixtures. The tests indicated that after the critical bottom velocity is exceeded, the capacity of transporting sand is directly proportional to the square of the bottom velocity. In connection with the possible application to conditions in natural streams, it is concluded that "for granular materials smaller than 5 mm, in a natural stream with surface slopes remaining approximately constant, du Boys' formula is applicable for determining the capacity of the stream".

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[22] "Bed Sediment Transportation in Open Channels", by C. H. MacDougall, *Transactions*, Geophysical Union, 1934.

[23] "Some Model Investigations of the Motion of the Sand Along a Self-Formed Channel," by H. Nakayama, *Journal*, Faculty of Engineering, Tokyo, Vol. 13–6, 1923.

[24] In unpublished communication with the writer.

[25] "Neue Grundlagen für die Berechnung der Geschiebeführung in Flusslaufen", von F. Schaffernak, 1922.

ESTIMATE OF BED-LOAD MOVEMENT IN A RIVER SECTION

The general formula (Equation (20)) derived previously can be applied to a natural stream for a rough estimate[26] of the rate of its transportation. Further, it has been shown that the critical tractive force is constant for a certain material, and that it may be expressed as

$$T_0 = 0.0175 \left[\frac{\sigma - \rho}{\rho} D\, o^{1/3} \right]^{\beta} = w\, S\, d_0$$

If the slope of the stream and the value of T_0 for its bed material are known, d_0 can be computed. In Fig. 12 (*a*), it is evident that motion and hence transportation of bed material will take place only at and below the depth, d_0—that is, on the surface, *C E D*—and that there is no motion on the sides, *O C* and *D F*. Actual observations by Kurzmann[27] support this theoretical consideration. The area, *A C E D B*, is thus called the effective area. Using the general formula and taking *O* as origin,

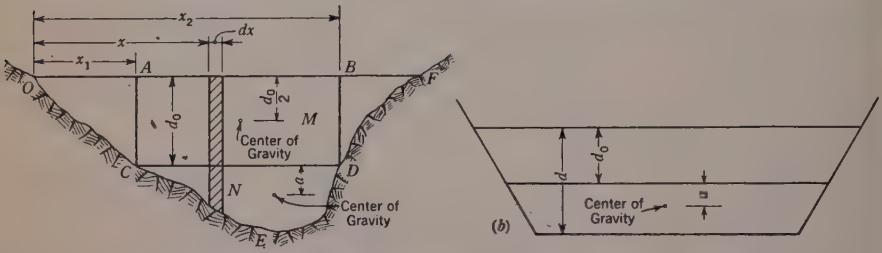

FIG. 12.—DATA ON TRANSPORTATION BY TRACTION: SAMPLE NO. 6197

$$G = \psi \int_{x_1}^{x_2} n\, T\,(T - T_0)\, dx = \psi \int_{x_1}^{x_2} n\, w^2\, S^2\, d(d - d_0)\, dx$$

$$= k \left[\int_{x_1}^{x_2} d^2\, dx - d_0 \int_{x_1}^{x_2} d\, dx \right]$$

The first integral is an expression of the moment of area about the surface. Dividing the total effective area into a rectangle, *A B C D*, called *M*, and an area, *C E D*, called *N*, the center of gravity of which is calculated and located at a distance $(d_0 + a)$ below the water surface,

$$2 \int_{x_1}^{x_2} (d\, dx)\, \frac{d}{2} = 2 \left[\frac{M\, d_0}{2} + N\,(d_0 + a) \right] = M\, d_0 + 2\, N\,(d_0 + a)$$

The second integral is simply an area $(M + N)$, and,

$$d_0 \int_{x_1}^{x_2} d\, dx = d_0\,(M + N)$$

Combining,

$$G = k\,[M\, d_0 + 2\, N\,(d_0 + a) - (M + N)\, d_0] = k\, N\,(d_0 + 2\, a) \ldots (44)$$

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[26] "Sand-Carrying Capacity of Streams", by Y. T. Sah, *Proceedings*, Eng. Soc., Tsing Hua Univ., Peiping, China, 1932.

[27] See "Wasserbauliche Strömungslehre," by P. Némenyi, 1933, p. 139.

from which it is seen that the quantity of sand moved is proportional to the area below the critical depth, d_0, times approximately the mean depth of the total effective area.

The application to a trapezoidal section (Fig. 12 (b)) is still simpler. Let A be the total effective area of the section; A_0, the upper trapezoid; and N, the lower trapezoid below the depth, d_0. Since a is approximately equal to $\frac{(d - d_0)}{2}$, then

$$G = \psi\, n\, w^2\, S^2\, N\, (d_0 + 2\, a) = \psi\, n\, w^2\, S^2\, d(A - A_0) = k\, d^{0.5}\, S^{1.5}\, (Q - Q_0) \quad \text{..(45)}$$

which approximates the general form of Equations (32).

SORTING PHENOMENON

In the present experiment on transportation, at the stage of initial dragging or rolling, it was observed that the coarser material was being dragged or rolled more rapidly than the finer. L. G. Straub,[28] Assoc. M. Am. Soc. C. E., noticed the same phenomenon in his experiments on flume traction. He observed that a surprisingly uniform gradation took place from one end of the flume to the other, despite the fact that the original material was of quite uniform size particles.

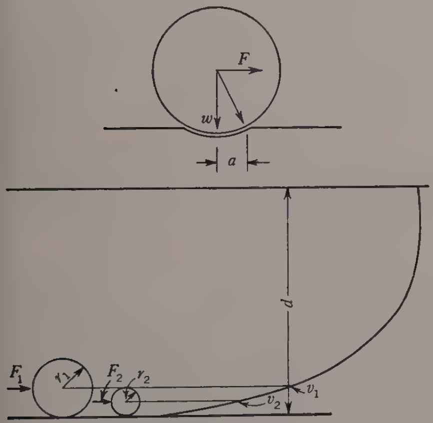

FIG. 13.—DATA ON TRANSPORTATION BY TRACTION: PUMICE

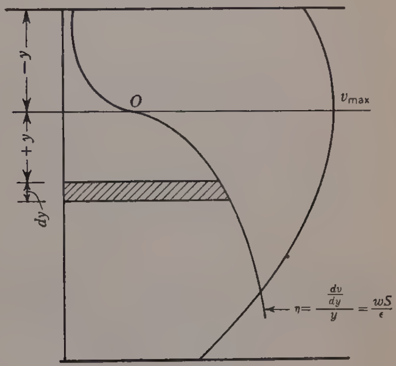

FIG. 14

The cause of this sorting is illustrated in Fig. 13, which shows two particles of different sizes on the bottom of the flume. The velocity distribution curve is horizontally exaggerated. Consider, first, the particles sliding on the surface. In turbulent flow the pressure on the particle is proportional to the velocity squared. By equating the pressure of the bottom velocity head on a spherical particle to the frictional resistance of the bed at impending motion,

$$\pi\, r^2\, w\, \frac{V_b^2}{2\, g} = f\, \frac{4}{3}\, \pi\, r^3\, (\gamma - w)$$

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[28] "Some Observations of Sorting of River Sediments," by L. G. Straub, *Transactions*, Am. Geophysical Union, 1933.

and

$$V_b = \left[\frac{4}{3} f\, 2\, g \frac{\sigma - \rho}{\rho} r\right]^{1/2} \propto \sqrt{r} \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

If the velocity distribution curve is of parabolic form, $v = k\, r\, (2\, d - r)$, and if, in addition, r is small compared with $2\, d$, $v \propto r$. If $r_1 = 2\, r_2$, the larger sphere will be moved by a velocity only 1.414 times as great as that required to move the smaller one. Actually, the velocity, v_1, at a distance, r_1, from the bed, is likely to be more nearly $2\, v_2$, for the velocity increases almost linearly near the bottom. Hence, the coarser particle is more easily and quickly moved than the finer one.

Next, consider particles rolling along. Equating the moment of the frictional resistance of the bed to that of the water pressure on a particle

$$\pi\, r^3\, w \frac{V_b^2}{2g} = \frac{4}{3} \pi\, r^3\, (\gamma - w)\, a$$

in which a, as shown in Fig. 13, is called the coefficient of rolling resistance and may be assumed, in the case of a soft bed, to be proportional to the radius of the particle, that is, $a = F\, r$. In this case, V_b is again expressed by Equation (46). Therefore, the sorting phenomenon is independent of the mode of movement; it exists either in sliding or in rolling.

In experiments on transportation of a graded sand, it is usually found that a large percentage of the coarser constituents is sorted out and transported. That this should occur can be demonstrated from the general equation,

$$G = \frac{k}{(D)^{2\beta}} T\, (T - T_0)$$

in which β varies from $\frac{1}{2}$ to 1 according to Equation (20). If the change in T_0 due to the change of D is negligible in comparison with T, then, approximately,

$$G = K \left[\frac{a}{(D_a)^{2\beta}} + \frac{b}{(D_b)^{2\beta}} + \frac{c}{(D_c)^{2\beta}} + \cdots\right] = K\, \Sigma \frac{p}{(D)^{2\beta}} \ldots\ldots (47)$$

in which a, b, c, $\cdots$, and p are the percentages by weight of the material transported. To have G a maximum, differentiate with respect to D, taking both G and p as variables, and equate to zero:

$$DG = K \left[-\frac{2\, \beta\, p}{D^{1+2\beta}} dD + \frac{1}{D^{2\beta}} dp \right] = 0$$

$$\frac{dp}{p} = 2\, \beta \frac{dD}{D}$$

or

$$\frac{p_1}{p_2} = 2\, \beta \left(\frac{D_1}{D_2} - 1\right) + 1 \ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Equation (48) is the general formula for sorting in sand transportation. Note, for example, that if the coarser particles are twice the diameter of the finer ones (that is, if $D_1 = 2\, D_2$ and $\beta = \frac{1}{2}$ for fine sands), then $\frac{p_1}{p_2} = 2$. In other words, two-thirds of the material transported will consist of coarse particles and one-third of fine particles.

SUMMARY: PART II

(1) An equation of the du Boys type is found to fit best the present experimental data. The specific form suggested is $G = \frac{K\,n}{T_0^2} T\,(T - T_0)$.

(2) The advance of riffles is proportional to the fifth power of the mean velocity of the flow.

(3) Sorting always takes place in graded sands, irrespective of the mode of motion of the particles, the coarser particles moving in greater quantities than the finer.

PART III.—TRANSPORTATION OF MATERIAL IN SUSPENSION

METHOD OF EXPERIMENT

Suspension was visible in the flume as soon as saltation began. When saltation was strong and accompanied by dune formation of the bed material, bands of particles were lifted vigorously and dispersed into suspension. The present observations were usually made under the latter conditions, and with the bed continuously changing, and are, therefore, only approximate. However, the results were consistent enough to illustrate the principles reviewed and developed in this part of the paper.

In addition to the data required in the study of transportation by traction, it was necessary in the present case to measure the velocity distribution of the section and to obtain samples of the suspended load. Pitot tube readings were taken at vertical intervals of about 1 in. in the axial plane of the flume. At the same time samples were taken at prescribed depths by means of a glass tube, $\frac{1}{4}$ in. in diameter, equipped with a suction bulb. These samples were transferred to a flask, in which the suspended particles soon settled to the bottom, and the concentration, in particles per cubic centimeter, was obtained by direct count of the particles and measurement of the volume of water. No attempt was made to investigate the size of suspended particles microscopically.

During these tests, feeding at the up-stream end was stopped to ensure that all the suspended load was supplied from the bed itself.

LIFTING FORCE NEAR BOTTOM

The lifting force on a particle resting on the bed may be considered as due mainly to the pressure of a velocity head. The particle will tend to lift when this vertical force just balances its effective weight in water. If V_l is the velocity of flow near the bottom required to produce this lift, the balance can be expressed mathematically by,

$$w \frac{V_l^2}{2\,g} \frac{\pi\,D^2}{4} = \frac{\pi\,D^3}{6} (\gamma - w)$$

whence

$$V_l^2 = \frac{4}{3} g \frac{\sigma - \rho}{\rho} D \ldots\ldots\ldots\ldots (49)$$

Comparing Equation (49) with Equation (46),

$$\frac{V_l}{V_b} = \frac{1}{\sqrt{f}} \quad \text{(50)}$$

in which f is the coefficient of friction between particles in water. The present experiments yield an average ratio of 1.4; this will correspond to $f = 0.51$. The value of f usually varies with ρ_e and D.

Gilbert observed in his studies of flume transportation that vortex rings with transverse axes occur near the bottom of the flume. Thus, the phenomenon of lifting can also be interpreted in a mathematical way[29] by "circulation." Since the radius of the vortex ring is so large compared with the size of the grain, the vortex ring may be considered as an imaginary large hollow cylinder. By the interrelated action of the horizontal velocity and the circulation, two equal forces are created to support the particle; one is due to the change of momentum and the other to the difference of velocity head above and below the particle. The final form of total lift is $\rho\, V\, \Gamma$, in which ρ and V are the density of the liquid and the velocity of flow, and Γ is the circulation, which is shown in aeromechanics, to be proportional to V. This agrees with the previous result that the lifting force is proportional to the square of the velocity.

DISTRIBUTION OF SUSPENDED SEDIMENT

The general equation[30] for distribution of suspended sediment is,

$$c = c_0\, e^{\omega \rho \int_0^y \frac{dy}{\epsilon}} \quad \text{(51)}$$

in which ϵ, defined by $-\dfrac{w\, S\, y}{\dfrac{dv}{dy}}$, is the Austausch coefficient; ω is the settling velocity of the solid particles; c is the sediment concentration at the elevation, y, and c_0 is the sediment concentration at the elevation of maximum velocity. If ϵ is a constant, Equation (51) can be expressed as

$$c = c_0\, e^{\omega \rho y / \epsilon} = c_0\, e^{\alpha y} \quad \text{(52)}$$

y being measured downward from the surface. The equation developed by Hurst[31] from experiments with sand stirred up into suspension by propelling the water in a cylinder, is of a similar form. He considers that in any layer, dy, the weight of the particles will balance the pressures due to the impact of particles from above and below. The result is:

$$c = c_0\, e^{\beta y} \quad \text{(53)}$$

in which $\beta y = \dfrac{3\, g}{\bar{u}^2} \dfrac{\sigma - \rho}{\rho} \displaystyle\int_0^y dy$, and $\bar{u}$ is the absolute mean velocity of the particles. Equations (52) and (53) show agreement in that both α and β

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[29] "Prandtl-Lanchester Theory", *Engineering*, Vol. 67, 1924.

[30] "Distribution of Silt in Open Channels", by J. E. Christiansen, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, 1935.

[31] "The Suspension of Sand in Water", by H. E. Hurst, *Proceedings*, Royal Soc., London, Vol. 124, 1929.

depend on the size and specific gravity of the grains and the turbulence of the flow. In the closing discussion on his paper entitled "Modern Conceptions of the Mechanics of Fluid Turbulence", Hunter Rouse,[32] Assoc. M. Am. Soc. C. E., has constructed a set of theoretical sediment distribution curves based on the assumption that the universal logarithmic velocity curve actually extends to the free surface of a wide stream. The general equation governing the sediment distribution is,

$$\frac{c}{c_a} = \left[\frac{y\,a}{(d-y)\,(d-a)}\right]^z \quad \text{.....................} (54)$$

in which $z = \dfrac{\omega}{k\,V\sqrt{\dfrac{f}{8}}}$. The equation is applicable down to some arbitrary depth $(d - a)$, beyond which the suspension becomes mixed with the tractive load. In the parameter, z, k is an universal constant having an empirical value 0.4 and f is the roughness coefficient of the channel.

Thus, a general conclusion can be drawn that whether the velocity distribution is parabolic or logarithmic, the vertical distribution of suspended sediment in turbulent streams is exponential.

EXPERIMENTAL VERIFICATION

Tests have been made on sediment distribution as described; the results plotted do not agree with the theoretical Equation (51), because this equation is presumably derived for a considerably wide channel suffering no side restriction. Assuming ϵ to be a constant (as it is for a parabolic velocity distribution), and S, the energy gradient, to be practically constant at any depth, y, then Equation (51) can be written in the form,

$$c = c_0\,e^{kw\,Sy} = c_0\,e^{\phi\,T}$$

which means that the sediment distribution is an exponential function of the tractive force at the depth. This is particularly in agreement with Bottomley's idea[33] that the silt transportation force depends mainly on the frictional resistance of the bed, or, in other words, the tractive force on the bed. The flume used in the present experiments is very narrow and it is evident that the depth, d, cannot take the place of the hydraulic radius, R, and, therefore, the exponent, $\omega\,\rho \displaystyle\int_0^{\blacksquare} \frac{dy}{\epsilon}$, must be adjusted accordingly. Using Equation (12), assuming an average value for d of $\frac{1}{2}$ ft, and taking $n_b = 2\,n_l$, the average tractive force becomes

$$\overline{T} = w\,S\,d\,\frac{1}{1 + \dfrac{2\,d}{B}\,\dfrac{n_l}{n_b}} = \frac{1}{2}\,T$$

Even this equation, however, will not apply exactly in the present case, because both the depth and the channel roughness vary continuously; hence, the exact correction is unknown. Several trials of plotting have shown, however, that the exponent, $\frac{\omega\,\rho}{2}\int\frac{dy}{\epsilon}$, fits the experimental data better than any other.

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[32] *Proceedings*, Am. Soc. C. E., January, 1937, p. 116.

[33] "A New Theory of Silt and Scour", by W. T. Bottomley, *Engineering*.

Before proceeding further, it is necessary to compute the settling velocity of sand in water. Rubey's formula,[34] reduced to a temperature of 15° C, is

$$\omega = \frac{7}{D}\left[\sqrt{134.5\,(\sigma - 1)\,D^3 + 1} - 1\right]$$

D being expressed in millimeters and ω in millimeters per second. The values of ω for pumice, Sand No. 6197, and emery are, respectively, 0.0886, 0.0853, and 0.1137 ft per sec.

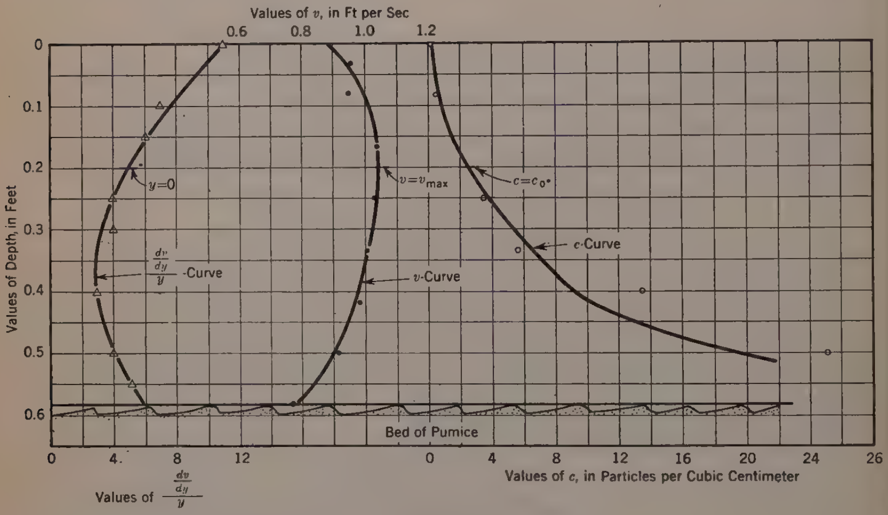

FIG. 15

The velocity curve is first plotted as shown in Fig. 14. The point of maximum velocity is taken as the origin, with positive values of y being measured downward therefrom. Values of $\frac{dv}{dy}$ are scaled directly from the curve, divided by the corresponding values of y, and plotted as the η-curve. The latter is indeterminate at the point of maximum velocity, but the upper and lower branches can be joined by a smooth continuous curve. The area of the η-curve to any co-ordinate, y, is equal to, $w\ s\int_0^y\frac{dy}{\epsilon}$, and can be measured by planimeter, the values always being taken as positive. From these graphical data, silt distribution curves can be computed once the specific concentration, c_0, and the value of ω are known. Curves for the various bed materials tested (see Fig. 15 for a typical example) were prepared on that basis. The velocity distribution curve was first plotted and the η-curve computed. Then, the observed concentrations were plotted and joined with a tentative curve, and the abscissa for $y = 0$ was taken as c_0. Theoretical values of c for other depths were then computed and plotted. If this curve did not agree with the original trial curve, the latter was modified accordingly—that is, varied to give a new value for c_0—and the computations were repeated.

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[34] "Settling Velocities of Gravel, Sand, and Silt Particles", by W. W. Rubey, *American Journal of Science*, Vol. 25, 1933.

TOTAL LOAD IN SUSPENSION

The following theoretical study assumes a parabolic velocity distribution, with maximum velocity at the surface (Fig. 16), and the silt concentration at the surface is taken as c_0. Then the mean value of the silt concentration throughout the depth is

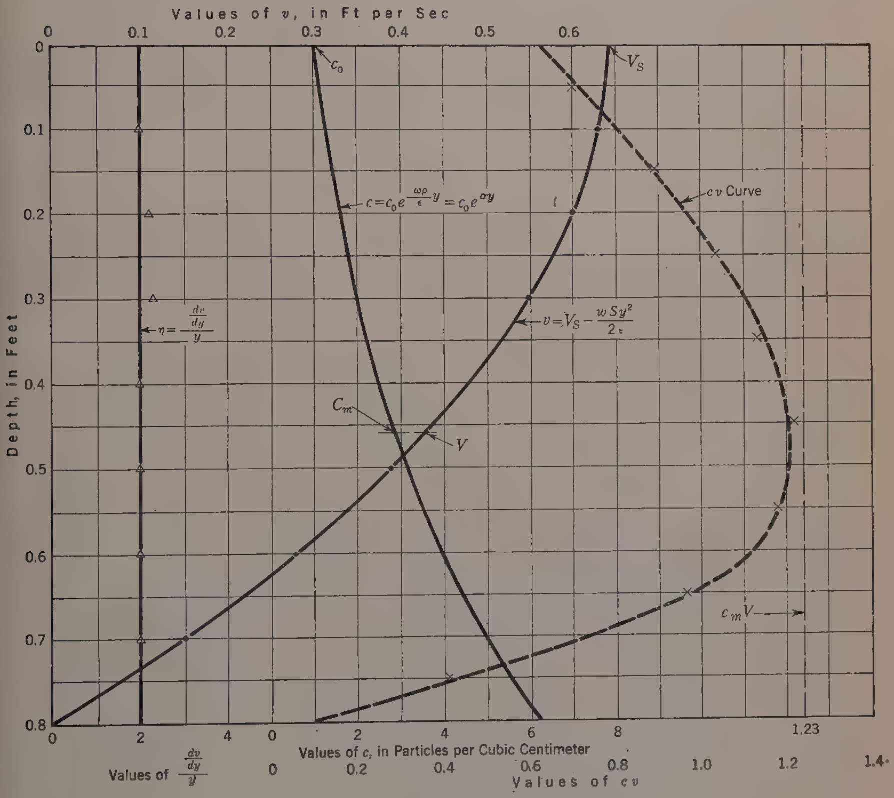

FIG. 16

$$c_m = \frac{1}{d}\int_0^d c\,dy = \frac{c_0}{d}\int_0^d e^{\alpha y}\,dy = \frac{c_0}{d}\left[\frac{e^{\alpha y}}{\alpha}\right]_0^d = \frac{c_0}{\alpha\, d}\left(e^{\alpha d} - 1\right) \ldots\ldots (55)$$

in which $\alpha = \frac{\omega\,\rho}{\epsilon}$. For the case plotted in Fig. 16, the following values have been assumed: $V_S = 0.64$, $d = 0.80$, $S = 0.00155$, $\omega = 0.0575$, and $c_0 = 1$; these are more or less the average conditions of the experiment

$$\omega\,\rho \int_0^y \frac{dy}{\epsilon} = \omega\,\rho \int_0^y \frac{\frac{dv}{dy}}{w\,S\,y}\,dy = \frac{\omega}{g\,S}\int_0^y \frac{\frac{dv}{dy}}{y}\,dy = \frac{\omega}{g\,S}\,\eta\,y = \alpha\,y$$

For a parabolic velocity distribution curve, η is a constant, its value in the present case being 2. Therefore, $\alpha = \dfrac{0.0575 \times 2}{32.2 \times 0.00155} = 2.303$; $\alpha\,d = 1.843$; and, substituting in Equation (55), $c_m = 2.88$ particles per cubic centimeter. Equating c_m to c to locate the depth of the point of mean concentration, $e^{\alpha y} = 2.88$; whence, $y = 0.46 = 0.575\,d$ from the surface.

Assuming that the vertical co-ordinates of the mean velocity and the mean concentration are nearly the same, the total suspended load, P, may be expressed approximately by,

$$P = c_m\,q = c_m\,V\,d = \frac{2\,c_0}{3\,\alpha}\,(e^{\alpha D} - 1)\,V_S \ldots\ldots\ldots\ldots\ldots (56)$$

in which c_0 is in pounds per cubic foot. Substituting the numerical values for an illustration, $P = 0.985$ (pound per second per foot of width).

If the velocity distribution is parabolic but the bottom velocity is not equal to zero, the following approximation may be used:

$$V = \frac{1}{d}\int_0^d v\,dy = \frac{1}{d}\int_0^d \left[V_S - \frac{w\,S\,y^2}{2\,\epsilon}\right] dy = \left[V_S - \frac{\eta\,d^2}{6}\right]$$

and

$$P = \frac{c_0}{\alpha}\,[e^{\alpha d} - 1]\left[V_S - \frac{\eta\,d^2}{6}\right] \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

Equations (56) and (57) do not give the exact total suspended load even when the co-ordinates of the mean velocity and the mean concentration are at the same elevation; the resulting value is always somewhat too high. However, the following exact solution may be applied to any cases as long as the velocity distribution is parabolic and the silt distribution exponential:

$$P = \int_0^d c\,v\,dy = c_0 \int_0^d e^{\alpha y}\left[V_S - \frac{\eta\,y^2}{2}\right] dy$$

$$= \frac{c\,V_S}{\alpha}\,[e^{\alpha d} - 1] - \frac{c_0\,\eta}{2\,\alpha^3}\,[(\alpha\,d)^2\,e^{\alpha d} - 2\,(\alpha\,d - 1)\,e^{\alpha d} - 2] \ldots (58)$$

This is the general equation for total load in suspension. c_0 is the concentration at $y = 0$; that is, at the surface where the velocity is a maximum. If $v = 0$ at the depth, d, $\dfrac{\eta}{2} = \dfrac{V_S}{d^2}$; therefore,

$$P = \frac{c_0\,V_S}{\alpha^3\,d^2}\,[2\,e^{\alpha d}\,(\alpha\,d - 1) - (\alpha\,d)^2 + 2] \ldots\ldots\ldots\ldots (59)$$

Substituting the numerical values, $P = 0.755$ (pound per second per foot of width). Curves of v, η, and c are plotted in Fig. 16. The ordinates of V

and c_m happen in this case to be at the same elevation. Their product gives the vertical line, $c_m V$; the area bounded by this line, the surface, and the axes, gives the total load of suspension by Equation (56). More accurately the product of c and v at various elevations should be plotted to obtain the curve, $c\,v$, the area to the left of which is the total load of suspension by Equation (59).

Straub[15] suggested the following (Equation (60)) derived from typical velocity-depth curves and typical suspended sediment-concentration curves, to determine the total quantity of silt transported in suspension per second:

$$P = \left[\tfrac{5}{8}\,c_{0.2d} + \tfrac{3}{8}\,c_{0.8d}\right] q \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

In the present theoretical case, $c_{0.2d} = 1.5$, and $c_{0.8d} = 4.3$, so that $P = 0.855$ (pound per second per foot of width) which is 13.2% greater than the value by the exact solution, and 13.2% less than the value by the approximate solution. For practical purposes Straub's empirical formula is probably of the right order. For a rough estimate of the quantity of silt transported by a river it is highly recommended, because it is fairly simple to make two soundings at the proper depths, to collect the silt concentration samples. However, for a theoretical parabolic distribution of velocity, Equation (57) is convenient and more accurate. Of course for a refined determination, an arithmetic integration of the product of V and c at numerous elevations is needed.

Effect of Silt Transportation on Velocity and Stream Energy

Transportation by traction affects the mean velocity of the stream by changing the roughness of the bed and, consequently, by varying the rate of dissipation of stream energy. Transportation by suspension, if not of appreciable quantity, will not affect the mean velocity very much. In rivers such as the Yellow River, however, with a maximum silt content of approximately 10% by weight during flood flows, and the Wei and Fen Rivers, tributaries of the Yellow River, with maximum silt contents as high as 46% and 20%, the suspended load will affect the mean velocity much more seriously than the bed load.

Evidently, the silt load in suspension will affect the mean velocity mainly in two respects: First, by changing the energy content of the stream; and, second, by changing the density and viscosity of the fluid. Changes in kinetic energy along the course of a stream due to the addition of silt are relatively small; further, little energy is consumed in supporting the débris. Therefore, it may be assumed that the change of kinetic energy due to the addition of silt is nil. Let $k\,w\,V\,Q$ be the kinetic energy of a clear stream. If V_1 is the mean velocity of the same stream with a quantity, $\alpha\,Q_1$, of silt in suspension, then the kinetic energy is represented by $k\,[w\,Q_1 + \alpha\,Q_1\,(\gamma - w)]\,V_1$. Equating the two expressions by assuming that the cross-sectional area remains the same, and solving for V_1:

$$V_1 = \sqrt{\frac{w}{w + \alpha\,(\gamma - w)}}\,V = \frac{V}{\sqrt{1 + \alpha\left(\frac{\sigma}{\rho} - 1\right)}} \quad \ldots\ldots\ldots (61)$$

Taking $\sigma = 2.65$, $\rho = 1$, and $\alpha = 10\%$ (for the Yellow and Colorado Rivers), $V_1 = \frac{V}{1.079}\cdot$ The velocity of flow is thus retarded by 7.4 per cent. For the Wei River with $\alpha = 46\%$, $V_1 = \frac{V}{1.326}$; that is, the velocity is decreased by 24.5 per cent.

This diminution of velocity of flow by silt content may serve to explain the "saturation" stage of stream capacity in supporting débris, for it is clear that silt will be transported in suspension only as long as the stream velocity exceeds the competent velocity to cause lifting up of particles.

Gilbert's observations[17] showed the same result. He concluded that under all conditions, streams are retarded by their suspended loads. The average of the thirteen velocities found for the loaded streams was 10% less than the corresponding average for unloaded streams.

In a paper published by Rubey[35] further light is thrown on this subject. By assuming that the change of kinetic energy is negligible, and equating the loss of potential energy of the flow (water + load) to the energy consumed in friction plus that consumed in transporting the load, he derived the following expression:

$$(W + L)\, g\, S\, V = \kappa\, \rho_f\, p\, V^4 + L \frac{\sigma - \rho}{\sigma}\, g\, \omega \ldots\ldots\ldots\ldots (62)$$

in which ρ_f is the density of flow (water + load); W and L are the weights of water and load, passing the section each second, respectively; κ, the skin friction coefficient of the channel; and p the wetted perimeter. When $L = 0$, $W\, g\, S\, V = \rho_f\, Q\, g\, S\, V = \rho_f\, A\, g\, S\, V^2 = \kappa\, \rho_f\, p\, V^4$; and $V = \left[\frac{g}{\kappa} R\, S\right]^{1/2}$, which is the familiar Chezy equation with its coefficient, C, expressed as $\left(\frac{g}{\kappa}\right)^{1/2}\cdot$ Substituting $C^2\, R\, S$ for V^2 in Equation (62),

$$\frac{\frac{L}{W}\,\frac{\sigma - \rho}{\sigma}\,\omega}{\left(1 + \frac{L}{W}\right) S\, V} = 1 - \frac{\kappa}{g} C^2 = \delta \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

When $L = 0$, $C = \sqrt{\frac{g}{\kappa}}\cdot$ When there is suspended load, C will be less than $\sqrt{\frac{g}{\kappa}}\cdot$ In the twenty-two experiments of Gilbert on débris-laden streams recalculated by Rubey, the values of $\left(1 - \frac{\kappa\, C^2}{g}\right)$ range from 0.007 to 0.070, with a median value of 0.025; that is, under the conditions of these experiments, an average of 97.5% of the total stream energy appears to have been lost in friction, and only 2.5% in transporting débris. In this equation $\frac{L}{W}$ is the fractional weight of the load in the stream. In the case of the Yellow River flood in 1931, with

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[35] "Equilibrium Conditions in Débris-Laden Streams", by W. W. Rubey, *Transactions*, Am. Geophysical Union, 1933.

39% loess suspension, δ was 0.0613 (taking $\sigma = 2.63$; $\omega = 0.004$ ft per sec, $S = 0.000135$, and $V = 84$ ft per sec).

Another substitution of the Chezy formula into Equation (63) gives,

$$\frac{\dfrac{\sigma - \rho}{\sigma}}{C\left(1 - \dfrac{\kappa}{g} C^2\right)} \times \frac{L\,\omega}{\rho_f\,Q} = S^{3/2}\,R^{1/2} \ldots\ldots\ldots\ldots\ldots (64)$$

Equation (64) states the general relationships that the graded slope varies directly as the quantity of débris that must be carried, directly as the average size of the material, and inversely as the discharge of the stream.

Let $X = \dfrac{\text{mean depth}}{\text{mean width}}$. Then $R = \dfrac{A}{p} = \dfrac{Q}{C\,\sqrt{R\,S}\,p} = \dfrac{Q\,X}{C\,R^{3/2}\,S^{1/2}}$, and $R^{1/2} = \left[\dfrac{Q\,X}{C\,S^{1/2}}\right]^{1/5}$. Substituting this in Equation (64),

$$S^{7/5}\,X^{1/5} = \frac{\dfrac{\sigma - \rho}{\sigma}}{C^{4/5}\left(1 - \dfrac{\kappa}{g} C^2\right)\rho_f} \times \frac{L\,\omega}{Q^{6/5}} \ldots\ldots\ldots\ldots\ldots (65)$$

In this expression the settling velocity,[34] ω, is proportional to $D^{1/2}$ for sand particles coarser than 1.55 mm in diameter, and to D^2 for sand particles finer than 0.145 mm in diameter. Consequently, the equation can be written in the form, $L \propto \dfrac{S^\alpha\,Q^\beta\,X^\gamma}{D^\phi}$.

Table 6 lists values of these exponents as obtained from several sources:

TABLE 6

Source	α	β	γ	ϕ
Theoretical from Equation (65)	1.40	1.20	0.20	0.50–2.00
Mean in Gilbert's empirical equation*	1.59	1.02	0.47	0.58
Rubey's result on Illinois River†	1.50	1.25	0.25	....
Gibson's experiment on Severn Model‡	....	1.00	....	0.77
Present result of the writer§	1.40	1.20	....	0.50–2.00

* "The Transportation of Debris by Running Water", by G. K. Gilbert, U. S. Geological Survey, *Professional Paper 86*, 1914.

† "Equilibrium Conditions in Debris-Laden Streams", by W. W. Rubey, *Transactions*, Am. Geophysical Union, 1933.

‡ "Construction and Operation of a Tidal Model of the Severn Estuary", by A. H. Gibson, H. M. Stationery Office, London, 1933.

§ By assuming that all tractive transportation is lifted in suspension, that is, $L = G$.

Summary: Part III

(1) Theoretically, the force required to lift a particle from the bottom of a stream is about 40% greater than that required to move it along the bed.

(2) The general equation for vertical distribution of silt in two-dimensional flow is of the form of Equation (51).

(3) The mean silt concentration throughout a depth, d, may be computed by Equation (55), in which $\alpha = \dfrac{\omega\,\rho}{\epsilon}$. It is located approximately at $0.575\,d$ from the surface.

(4) The total suspended load in a stream may be well estimated by the empirical formula, Equation (60). Equation (58) is an exact equation, assuming a parabolic distribution of velocities.

(5) Silt transportation tends to decrease the mean velocity of flow. However, the energy consumed in supporting the load is only a small fraction of the total energy of the stream, which is mainly dissipated in overcoming the frictional resistance of the channel.

Acknowledgments

The writer is indebted to Professor A. H. Gibson for his advice and interest in the problem, and for offering every possible facility in the use of the Whitworth Engineering Laboratory of Victoria University, at Manchester, England. Mr. J. Allen helped to perform the experiment and corrected the original manuscript. Mr. S. D. Chyn, of Tsing Hua University, Peiping, China, read the paper and offered helpful criticism. The writer wishes also to express his sense of indebtedness to the Board of Trustees, Nanking, China, for the scholarship which enabled him to carry out the research.

APPENDIX

Nomenclature

The following nomenclature is used throughout the paper:

A = area of cross-section, in square feet;
a = vertical distance from d_0 to center of gravity of N; also, an exponent;
B = surface width, in feet;
C = Chezy coefficient; also, a constant;
c = constant in tractive force equations; elsewhere, concentration of suspended sediment, in particles per cubic centimeter; or in pounds per cubic foot;
D = mean diameter of a sand particle, in millimeters;
d = depth in feet; d_0 = depth at which critical tractive force obtains;
e = base of natural logarithms;
f = coefficient of friction in water;
G = rate of sand movement, in pounds per second;
g = gravity acceleration, in feet per second per second;
i = slope of side of trapezoidal channel, horizontal over vertical;
k = a coefficient;
L = total weight of suspended load passing a given section in 1 sec;
M = Kramer's uniformity modulus (dimensionless); also, area of channel above d_0;
N = area of channel below d_0;
n = Manning's n; also number of layers of sand (in explanation of du Boys' formula);
o = ratio between longest and shortest diameter of a sand grain;
P = total suspended load, in particles per second per centimeter of width; or in pounds per second per foot of width;

p = wetted perimeter; also, percentage of material of a given size in a mixture of sand that has been transported by traction;

Q = discharge, in cubic feet per second;

q = discharge per unit width of channel;

R = hydraulic radius, in feet;

$\mathsf{R} = \frac{V\,d}{\nu}$ = Reynolds number (dimensionless);

r = radius of sand particle;

S = slope of water surface; S_0 = slope of bed;

T = unit tractive force, in pounds per square foot; $\overline{T}$ = average unit tractive force over entire wetted perimeter; T_0 = critical unit tractive force;

$\bar{u}$ = absolute mean velocity of suspended particles;

V, V_1 = mean velocity, in feet per second; V_s = surface velocity; V_b = bottom velocity; and V_l = lifting velocity;

v = velocity at any point, in feet per second; v_r = advance of riffles, in feet per second;

W = total weight of water passing a given section in 1 sec;

w = weight of water, in pounds per cubic foot;

X = ratio of mean depth of channel to mean width;

x = length measured along the bed;

y = vertical distance, variable;

z = ratio between length of sloping side and bottom width of trapezoidal channel;

α = angle between water surface and horizontal; also, coefficient in Schoklitsch equation; also, an exponent; in Part III, $\alpha = \frac{\omega\,\rho}{\epsilon}$;

α_0 = angle between bed and horizontal;

β = an exponent to the expression $[\rho_e\,D\,o^{1/3}]$;

γ = unit weight of sand; also, an exponent;

$\partial = \left[1 - \frac{\kappa}{g} C^2\right]$, in Equation (63);

ϵ = Austauch coefficient $= -\frac{w\,S\,y}{\frac{dv}{dy}}$;

$\eta = \frac{\frac{dv}{dy}}{y}$;

κ = coefficient of skin friction (dimensionless);

μ = viscosity of fluid;

ν = kinematic viscosity of fluid;

ξ = height of riffle, in feet;

ρ = specific gravity of fluid; ρ_e, effective specific gravity of submerged sand particles; ρ_f, the specific gravity of flow (water + load);

σ = specific gravity of sand particles;

ω = settling velocity of sand particles in water, in feet per second.

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PAPERS

INCREASING THE TRAFFIC CAPACITY AND SAFETY OF THOROUGHFARES

A SYMPOSIUM

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NOTE.—The Symposium on Increasing the Traffic Capacity and Safety of Thoroughfares was presented at the meeting of the City Planning Division, Pittsburgh, Pa., October 15, 1936. Discussion on this Symposium will be closed in **March, 1938,** *Proceedings.*

CAUSES OF FAILURE IN HANDLING TRAFFIC

BY LEWIS W. MCINTYRE,[1] M. AM. SOC. C. E.

SYNOPSIS

The Bureau of Traffic Planning of the City of Pittsburgh, Pa., has conducted observations on down-town intersections to determine the space between moving automobiles, and also the time taken by automobiles to clear the intersection. Interference was at a minimum during these observations. The theoretical maximum traffic flow of a single lane of traffic was computed from these data. It is the purpose of this paper to show how various factors reduce the theoretical maximum traffic flow to the much lower values that occur in actual practice, in the usual congested district, and as far as possible to evaluate, quantitatively, the effect of each factor.

DEFINITIONS

The following definitions, essentially as proposed in 1934 by a Committee of the Institute of Traffic Engineers,[2] have been adopted for use in this paper:

(1) Traffic consists of pedestrians, mounted or herded animals, vehicles, street cars, and other conveyances together with their load, either singly or as a whole, while using any "way" for the purpose of transportation or travel.

(2) Traffic flow is the number of elemental components of traffic that pass a given point in a specified direction during a specified period of time (usually 1 hr). The traffic flow in an individual lane of traffic is determined by the effective headway and the speed.

(3) Traffic volume is the total number of elemental components of traffic that pass through a given intersection or way in any specified period of time (usually 8 hr, or more).

(4) Traffic capacity is the total number of elemental components of traffic that can pass through a given intersection or way in any specified period of time (usually 1 hr). The traffic capacity of any street depends upon the number of lanes of moving traffic on that street and the traffic flow in each individual lane.

(5) Traffic delay is the time spent while the elemental components of traffic that are being studied cannot move because of interference.

(6) Traffic speed is the rate of movement with which specified elemental components of traffic move over a specified course or section of way during any specified period.

(7) Over-all speed is the total distance traversed, divided by the total time required (usually expressed in miles per hour and includes all delays).

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[1] Cons. Engr., Pittsburgh, Pa.

[2] Rept. of Committee on Terminology and Nomenclature, Inst. of Traffic Engrs., T. M. Matson, *Chairman*. Not published.

(8) Weaving is the movement of vehicular traffic in such manner that the elemental components cross each other's paths at low angles of approach or in nearly the same direction of movement.

(9) A point of conflict is the point or limited area where elemental component parts of traffic intersect; the apex of the angle of approach.

(10) A lane of traffic movement is a series of elemental components of traffic moving, or standing ready to move, so as to use a lane of roadway effectively.

(11) A critical lane is a line of traffic which, for inherent reasons, requires more time to pass a given point than other parallel lanes.

(12) The lane width of a roadway is the measure of the number of possible free moving lanes of traffic which the roadway can pass in both directions: (*a*) A lane is a width of roadway sufficient to accommodate the free movement of an average vehicle, customarily considered to be 10 ft.

(13) Headway is the time space between the front ends of vehicles or street cars when moving past a specified point in a given direction and lane.

(14) Minimum headway is the time space (see Definition (13)) obtained when the elemental components of traffic are moving as closely together as practicable.

(15) Effective headway is the time space (see Definition (13)) obtained for elemental components of traffic moving under minimum headway in one or more lanes.

(16) A way, as applied to traffic, is that upon which traffic passes. The term, "way," shall be considered generic to all places along which traffic may move. It includes highway, parkway, limited way, freeway, road, path, route, street, avenue, boulevard, lane, alley, arcade, drive, etc.

(17) A major street is a way that carries traffic between sections of an urban district. The major streets of a city are the main traveled ways. They may form important connecting links in the traffic facilities of the State. (A major street is sometimes referred to as an arterial street.)

(18) A minor street is a way that connects individual property with a major street or other minor streets.

(19) An alley is a narrow inner street, either without sidewalks, or with very narrow ones, or a street on which the rears of buildings adjoin. It serves as a connection with other minor streets.

(20) A radial street is a major street that runs from the central business district to the outlying districts of a city. A radial street signifies a way that carries traffic from surrounding territory to a main focal point. The term is also applied to rural highways.

(21) A distribution street is a major street that collects and passes traffic from the other major streets and from which traffic distributes itself to advantageous points of entry into a district.

(22) A by-pass street is a major street that carries traffic around congested districts (sometimes referred to as a belt line or loop).

(23) A cross-town street is a major "way" that carries traffic so as to intersect radial streets at approximately right angles. (A cross-town street is also referred to as circumferential street.)

(24) A roadway is that part of a way particularly devoted to vehicular traffic; ordinarily, it lies between the regularly established curb lines.

(25) The pavement is that part of a roadway having a constructed surface for the facilitation of vehicular traffic.

(26) An intersection is the area of a junction included within lines joining the meeting points of property lines (together with any sections of property lines necessary to enclose the area) at the junction of two or more ways at the same elevation.

(27) A congested district is any area in which most of the streets or ways carry such a volume of traffic during a considerable part of the day as materially to reduce the headway and cause traffic delay (usually a business district).

TABLE 1.—Space Between Moving Automobiles at Intersections

No. of intersections	No. of observations	Average space, in feet	Remarks
3	574	17.82	Free movement
1	139	17.42	Following street car
1	71	12.8	Especially dense traffic

The spacing between vehicles (Table 1) was obtained by marking, on the street pavement, various distances from the property line. An observer recorded the distance of the rear end of each vehicle from the property line at the instant when the front end of a following vehicle passed this line. Under ordinary circumstances of fairly congested traffic, the average spacing between vehicles was 17.82 ft. When these vehicles followed street cars the space was 17.42 ft. In a number of cases where traffic was especially dense and was moving under obvious pressure, the spacing was as low as 12.8 ft. The data in Table 2 indicate that the first car of a group moved across the 36-ft roadway and one 12-ft sidewalk at an average speed of 10.8 or 10.9 miles per hr, depending on whether the vehicle was a passenger car or a truck. The data also indicate that following cars, because of their greater opportunity to get started, actually proceeded through the intersection at a speed of 15.3

TABLE 2.—Time Taken by Automobiles to Clear Intersection (12-Foot Sidewalk Plus 36-Foot Roadway)

Type of car	No. of observations	Time, in seconds	Speed, in miles per hour	Effective free movement, in seconds	Headway, dense traffic, in seconds	Lane Capacity (Theoretical)	
						Dense traffic	Free movement
Passenger:							
First car group.........	93	3.92	10.8	2.01	1.70	2 100	1 790
	161	2.77	15.3	1.43	1.20	3 000	2 500
Truck:							
First car group.........	21	4.61	10.9	2.78	2.44	1 475	1 300
	10	4.06	12.4	2.42	2.14	1 680	1 480
Mixed group...........	199	2.95	15.3	1.60	1.38	2 600	2 250

miles per hr in the case of passenger cars or a mixed group, and 12.4 miles per hr in the case of trucks. Using these speeds and the average spacing shown in Table 1, effective headways were computed under the conditions of reasonably free movement and of especially dense traffic. In these computations the length of a passenger automobile was taken as 14 ft and the length of a truck as 26 ft.

These data indicate that a single lane of a street in a congested district has a capacity of from 1 800 to 2 500 passenger cars per hr at an intersection, with interferences eliminated, and that reasonable speeds range from 11 to 15 miles per hr. The data also indicate that such a street has a capacity for trucks of from 1 300 to 1 500 vehicles per hr at speeds from 11 to 12.5 miles per hr. Where the traffic is mixed, as is usually the case, the capacity of a single lane is indicated to be approximately 2 250 vehicles per hr. Naturally, this capacity will depend upon the proportion of trucks and other vehicles present. The data further indicate that where traffic is especially dense, even higher values than the foregoing may be obtained. It is not likely, however, that these especially dense conditions could be maintained over a considerable period of time.

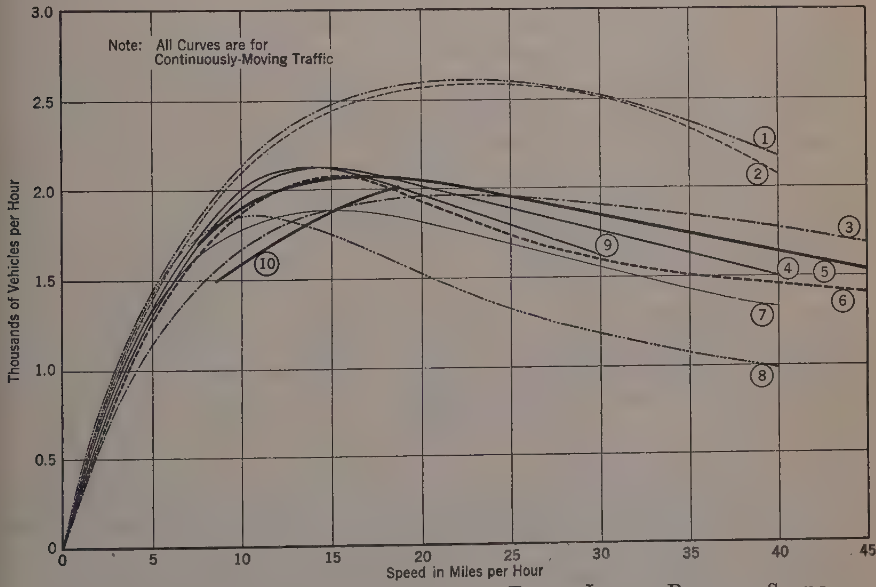

FIG. 1.—MAXIMUM HOURLY CAPACITY OF A SINGLE TRAFFIC LANE AT DIFFERENT SPEEDS

Some verification of these data was obtained from a group of curves (Fig. 1) obtained from various sources, which represent maximum hourly capacity of a single traffic lane at varying speeds. These curves show a range of hourly capacity of from 1 700 to 2 300 vehicles at 11 miles per hr, and from 1 900 to 2 500 vehicles at 15 miles per hr. Referring to Fig. 1, Curves 1, 4, and 9, respectively, are for Los Angeles, Calif., Detroit, Mich., and Norristown, Pa.[3]

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[3] "A Study of the Traffic Problem," by William S. Canning, 1928, p. 16. The curve for Los Angeles (Curve 1) is shown in more detail in *Traffic Bulletin No. 143*, Am. Gas Accumulator Co., Elizabeth, N. J.

Curve 2, originally, was plotted from computations made on the basis of field data referring to the spacing of vehicles at various speeds both on crowded and on high-speed thoroughfares.[4] It applies to a continuously moving lane of traffic, on the assumption that the average length of vehicle is 14 ft. Curve 3 was plotted from data observed by the Pennsylvania State Highway Department. This curve is intended to show the "theoretical maximum capacity of one traffic lane under ideal conditions."[5] The sources of the data, study conditions, assumptions, etc., from which the computations were made, are not available.

Curve 5, Fig. 1, was derived and plotted by the Bureau of Traffic Planning, City of Pittsburgh, and represents the mean average of the nine remaining curves in this drawing. A blueprint of Curve 6, with its formula, supplied by N. W. Dougherty, M. Am. Soc. C. E., was the only item of reference material available for this curve. Evidently, it applies to continuously moving traffic, assuming all cars with four-wheel brakes, and, being derived from an equation, is theoretical.

Curve 7 is intended for an "uninterrupted traffic lane."[6] It was derived from a formula, the factors of which were obtained from actual field data and observations. Mr. H. S. Swan,[7] who is the authority for Curve 8, makes no mention of study conditions, how data were obtained, etc., except that "safe braking distance" is assumed and that no time is allowed for the application of brakes. Two other curves presented by Mr. Swan (not included in Fig. 1) are computed on different assumptions. Finally, Curve 10 was plotted by the Pittsburgh Bureau of Traffic Planning, based on studies of down-town Pittsburgh. This curve is theoretical, representing the flow of traffic, for a full hour, moving in a flexible, progressive, signal system.

Further light on the accuracy of Fig. 1 was obtained from studies of the vehicle flow on the Wilmot Street Bridge, in Pittsburgh (see Table 3). This is a two-lane bridge—one lane in each direction. It has a large reservoir at one end for entering vehicles to accumulate, and at the other end for leaving vehicles to spread out promptly, so as not to delay those on the single lane of movement on the bridge. Table 3 shows an average rate of 1 710 vehicles per hr at a speed of 20 miles per hr, which rate was sustained over a period of 5 min at a time. A rate of 1 680 vehicles per hr was sustained for as long as 25 min.

Further supporting data (see Fig. 2) were obtained based upon a number of studies in down-town Pittsburgh, principally on Second Avenue, where dense groups of traffic were observed moving through the progressive traffic signal system. These data indicate a movement of 1 700 vehicles per hr at a speed of 11 miles per hr, and 1 900 vehicles per hr at a speed of 15 miles per hr. Fig. 2 is a comparison of Curves 3 and 5 in Fig. 1. Both curves are on the basis of a cycle of 53 sec and a lane-moving, or "Go" time of 45.3% of the cycle, or 24 sec. It is to be noted that, at the points of greatest divergence of the two, Curve 5 differs from Curve 3 by only 15% at 12 miles per hr and 19% at 17 miles per hr. These data give substantial ground for the traffic capacity of a single lane of a street in a congested district as 2 000 to 2 500 passenger cars per hr with speeds of movement of from 11 to 15 miles per hr.

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[4] "Roadway Capacity," by W. D. Hudson, July, 1924. Not published.

[5] "Facts and Figures of the Automobile Industry," National Automobile Chamber of Commerce, 1928 Edition, p. 85.

[6] "Regional Survey of New York and Its Environs," 1927 Rept., Vol. III, "Highway Traffic," p. 86, Fig. 45.

[7] "Theoretical Capacity of Roadways," by H. S. Swan, *Traffic Bulletin No. 142*, Am. Gas Accumulator Co., Elizabeth, N. J.

TABLE 3.—VEHICLE FLOW ON WILMOT STREET BRIDGE, PITTSBURGH, PA. (ONE LANE)

Date (1933)	No. of vehicles in 25 minutes	No. of vehicles in a maximum 5 minutes	Rate of flow, in vehicles per hour	Speed, in miles per hour
October 5.............	701	145	1 740	20
October 6.............	619	142	1 680	20
Average..............	143.5	143.5	1 710	20

Of course, these capacities are not actually obtained for all lanes, or even for the critical lane. Counts of actual traffic volumes and the hourly rate per lane for several typical congested Pittsburgh streets (Table 4), show a range of from 132 cars per hr to 754 cars per hr per lane. The highest rate (754 cars per hr per lane) was obtained on the Boulevard of the Allies at Grant Street, which is a viaduct approach to the congested district. The next highest rate of 508 was obtained on a bridge approach. At both these locations many of the usual interferences were not present, and there was constant pressure on the intersection from a long reservoir of automobiles waiting to get through.

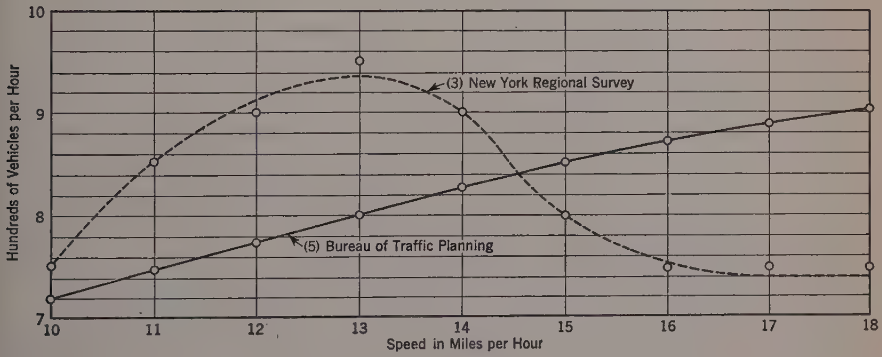

FIG. 2.—COMPARISON OF LANE CAPACITY DATA

The reason for this startling difference between capacities and actual flow is interference of one kind or another. Experience has shown that considerable interference can be eliminated, or at least reduced. Four types are recognized: (1) Cross-interference; (2) marginal interference; (3) internal interference; and (4) medial or center-line interference.

Interference by cross-traffic is the most obvious. It is evident that traffic from the main streets must stop while traffic on the cross-streets is moving.

If both the main street and the cross-street are given an equal proportion of the available time, it is obvious that the flow on each cannot be more than 50% of the capacity. Although this interference by cross-traffic can only be eliminated entirely by complete grade separation, it can be reduced by accurate proportioning of signal timing so that each street is given its proper share, but no more and no less than its proper share, of the available time. This proportion will vary throughout the day, but study indicates that the variation occurs with reasonable regularity, both as to time of the day, day of the week, and season of the year. Naturally, the facts concerning these variations must be obtained if the street is to be used at its maximum efficiency.

TABLE 4.—TRAFFIC VOLUME PER LANE, TYPICAL PITTSBURGH INTERSECTIONS

Street	At:	Traffic from:	Volume maximum, in 15 minutes	No. of lanes	Traffic per lane	Hourly rate per lane	Remarks
Fifth Avenue	Smithfield Street	East	124	2	62	248	Street cars
Fifth Avenue	Smithfield Street	West	195	2	98	392	Street cars
Ninth Street	Liberty Street	North	173	2	86	346	Street cars
Sixth Avenue	Grant Street	East	261	2	130	520	Street cars
Sixth Street	Penn Avenue	North	135	2	67	270	Street cars
Allies*	Grant Street	East	377	2	188	754	No street cars
Allies*	Grant Street	West	305	3	102	408	No street cars
Grant Street	Allies*	North	217	3	72	288	Street cars
Grant Street	Allies*	South	169	3	56	224	Street cars
Grant Street	Seventh Street	North	207	3	69	276	No street cars
Grant Street	Seventh Street	South	252	3	84	336	Street cars
Seventh Avenue	Grant Street	East	233	3	78	312	No street cars
Seventh Avenue	Grant Street	West	241	3	80	320	Street cars
Fourth Avenue	Smithfield Street	West	100	3	33	132	One-way†
Smithfield Street	Water Street	South	254	2	127	508	Bridge
Penn Avenue	Sixth Street	East	185	4	46	185	One-way
Smithfield Street	Fourth Avenue	South	196	4	49	196	One-way
Smithfield Street	Fifth Avenue	South	205	4	51	205	One-way

* Boulevard of the Allies. † Three lanes.

Cross-interference shows further restrictive effect in the delay that follows the stopping of a vehicle. Some data showing the time taken to start automobiles at an intersection, after the "Go" signal has been given, have been obtained as follows:

Position of car in line	No. of observations	Average time, in seconds
First car	224	1.6
Second car	224	2.9
Third car	202	4.1
Fourth car	185	5.1
Fifth car	101	7.4
Sixth car	81	7.6

The average time required to start equals approximately 1.41 times the position of the car in line. Naturally, the more stops that are caused by intersections which are too close together, the more frequently will this delay occur and the greater will be its effect in reducing street capacity. One of the fundamentals of good traffic regulation is to keep the traffic moving. The following simple illustration, involving the sixth and last car of a line stopped at an intersection, will show the importance of effective headway (rate of flow, 1 275 cars per hr):

Description	Time required, in seconds
For sixth car to start	7.6
To travel 48 ft, at 10.8 miles per hr, after starting	3.92
To travel 48 ft, at 15.3 miles per hr, through the intersection	2.77
To travel another 51 ft, at 13.0 miles per hr	2.67
Total time	16.96
Effective headway	2.82

Assume the six cars to be stopped with a distance of 3 ft between each car, and each car to be 14 ft long. The rear of the sixth car will be 99 ft from the intersection, and must travel 147 ft for the rear of the car to clear the far curb of a 36-ft roadway. The data in Table 2 demonstrate that the first passenger car of a group will travel 48 ft in 3.92 sec and that the following cars of this group will travel 48 ft in 2.77 sec. The speed of the first car in traveling 48 ft was 10.8 miles per hr and that of following cars, as they cleared the intersection, 15.3 miles per hr. Assuming the average of these two speeds as the speed which the sixth car made in the remaining 51 ft which it was required to travel, gives a total of 17 sec before this sixth car clears the intersection, or an effective headway for these 6 cars of 2.82 sec, and a rate of flow of 1 275 cars per hr per lane. This is a reduction in lane capacity of 35 to 50 per cent.

If intersections are close together there is no opportunity for vehicles to have sufficient free running time at higher speeds to overcome this delay, and the capacity of the street is permanently reduced.

A second cross-interference of considerable magnitude is caused by pedestrians who cross against the traffic. The degree of this interference depends upon the degree to which pedestrians obey the signals. Some data taken at typical intersections in the down-town district in Pittsburgh show that 56% of the pedestrians confronted by a "red" signal, failed to heed it, and crossed the street. The best obedience in the down-town district was at Smithfield Street and Sixth Avenue where 34% of the pedestrians confronted with a "red" signal crossed in the face of it. The corresponding value for Penn Avenue and Sixth Street was 71 per cent. An attempt was made to establish the relationship between pedestrian volumes, vehicle volumes, or turning volumes, and the degree of pedestrian observance of the traffic signals; but although a large volume of data was obtained, no consistent relationship could be determined.

In considering this pedestrian interference, it should be noted that no real effort to secure pedestrian control, except through educational means, has been attempted in Pittsburgh. Such interference, as well as that caused by pedestrians crossing in the middle of the block (commonly called "jaywalking"), has been reduced materially in a number of cities where real efforts have been made to enforce pedestrian regulations in congested districts. The value of such enforcement in increasing street capacity can be seen when it is realized that observations covering 4 hr for each of twelve intersections in down-town Pittsburgh showed that 22.9% of all pedestrians at the intersection crossed with interference of one sort or another. Naturally, the degree to which pedestrian interference reduces the street capacity depends on the volume of pedestrians present. Where such a large proportion crosses against the traffic the conclusion is inevitable that the street capacity has been materially reduced.

TABLE 5.—DELAYS CAUSED BY TURNS

TYPE OF TURN		Type of vehicle	Number of observations	TIME REQUIRED, IN SECONDS:		DELAY, IN SECONDS, DUE TO INTERFERENCE WITH ONCOMING:			DELAY, IN SECONDS, IN ITS OWN LANE, AT STRAIGHT-AHEAD SPEEDS (IN MILES) PER HOUR) OF:			
From Lane No.:*	To Lane No.:*			To clear its own lane	To turn	Lane No. 3*	Lane No. 2*	Lane No. 1*	6	12	18	Actual
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	1	Passenger	18	2.5	4.5	...	...	...	0.0	1.0	1.5	0.9
		Trucks	10	2.8	5.0	...	...	...	0.0	0.6	1.4	0.6
		Horse-drawn	1	6.0	8.8	...	...	...	2.4	4.2	4.8	1.2
1	3	Passenger	76	3.2	5.2	...	...	...	0.0	1.3	1.9	1.1
		Trucks	16	4.1	6.5	...	...	...	0.0	1.5	2.4	1.4
1	4	Passenger	22	3.0	5.0	...	...	...	0.0	0.0	0.9	0.0
		Trucks	16	3.2	6.3	...	...		0.0	0.0	0.7	0.0
		Horse-drawn	2	3.3	6.5	...	...	‡	0.0	0.0	1.0	0.0
2	2	Passenger	65	2.8	5.7	...	...	3.1	0.0	1.3	1.8	1.2
		Trucks	18	4.2	6.6	...	...	4.2	0.0	2.0	2.8	2.0
		Horse-drawn	3	4.9	7.6	...	...	5.4	1.3	3.1	3.7	0.1
2	3	Passenger	103	3.3	5.6	...	...	3.2	0.3	1.8	2.3	1.7
		Trucks	26	3.9	6.8	...	...	4.2	0.0	1.7	2.5	1.7
		Single street car	51	7.0	10.2	...	...	7.7	0.5	3.8	4.8	2.3
		Street-car train	14	12.7	22.6	...	...	13.5	0.9	6.8	8.8	4.2
		Horse-drawn	2	5.5	10.9	...	...	6.8	1.9	3.7	4.3	0.7
2	4	Passenger	45	3.4	5.5	...	...	3.1	0.4	1.9	2.4	1.8
		Trucks	29	3.3	6.8	...	...	3.2	0.0	1.1	1.9	1.1
		Horse-drawn	4	8.9	16.2	...	...	8.2	5.3	7.1	7.7	4.1
3	2	Passenger	167	4.0	6.9	...	2.7	2.7	1.0	2.5	3.0	2.4
		Trucks	30	5.6	9.1	...	4.3	4.5	1.3	3.4	4.2	3.4
		Single street car	46	8.2	12.0	...	8.4	8.7	1.7	5.0	6.0	3.5
		Street-car train	30	12.4	16.4	...	12.2	11.9	0.6	6.5	8.5	3.9
		Horse-drawn	1	5.2	9.5	...	5.4	5.2	1.6	3.4	4.0	0.4
3	3	Passenger	17	3.2	5.7	...	5.0	1.8	0.2	1.7	2.2	1.6
		Trucks	17	4.9	6.2	...	4.5	2.1	0.6	2.7	5.5	2.7
		Horse-drawn	1	7.0	12.2	...	5.2	4.8	3.4	5.2	5.8	2.2
4	2	Passenger	21	8.2	11.1	1.2	3.0	2.5	4.3	6.3	6.9	6.1
		Trucks	16	8.7	11.6	3.7	5.3	4.5	3.5	6.1	7.0	6.0
		Horse-drawn	1	4.2	9.4	...	4.4	5.2	0.0	1.9	2.7	0.0
1	1†	Passenger	20	2.3	3.8	...	...	...	0.0	0.8	1.3	0.7
		Trucks	7	2.6	4.5	...	...	...	0.0	0.0	1.2	0.4
2	2†	Passenger	90	2.4	4.3	...	...	2.8	0.0	0.9	1.4	0.8
		Trucks	29	2.7	4.9	...	...	3.2	...	0.5	1.3	0.5
		Single street car	21	8.2	11.4	...	...	11.1	0.1	4.2	5.5	2.4
		Street-car train	31	14.7	16.5	...	...	15.3	1.3	8.0	10.2	5.0
3	2†	Trucks	18	5.9	9.7	...	4.4	4.9	1.6	3.7	4.5	3.7

* All left turns, except as indicated in Column (2). † Right turn. ‡ Actual speed.

Turns are a further type of cross-interference, particularly left turns. Factual data regarding the magnitude of such interference have been obtained.

These data, in Table 5, show: (*a*) The time required by vehicles of various types to clear their own lanes; (*b*) the delay to following traffic; (*c*) the time required to complete the turn; and, (*d*) the delay to oncoming traffic where the turn crosses such traffic.

Turns are designated by the number of the lane from which, and to which, the turn is made. Lanes are numbered across both streets starting from the corner about which the turn is made. Average delays to following traffic range from zero to 6.1 sec, depending on the type of turn, whether the streets are one-way or two-way, the type of vehicle, the volume of oncoming traffic, etc. Interference to oncoming traffic ranges to as high as 15.3 sec, and time to complete the turn to as much as 22.6 sec in the case of a street-car train making the ordinary left turn from one two-way, four-lane street to another. Table 5 shows that one passenger automobile, turning left from the center lane of a two-way, six-lane street, may delay following traffic in its own lane an amount equal to the time required for four such vehicles to go straight through; and that the street-car train making the turn previously described, occupies the time required for twenty-one passenger cars to move through one of the lanes in the opposite direction.

Of course, the actual reduction in capacity caused by these movements at any given intersection depends on the number of such turns per hour and the actual traffic volumes moving in both directions. The data can only be used in a practical manner when these volume data are known from actual traffic counts at the points being studied.

In considering these and other data presented in this paper, it should be borne in mind that they are presented merely with the idea of evaluating the various interferences in terms of their reduction of street capacity and so that the value of providing remedies for these interferences can be measured. The economic phase has not been discussed; for instance, it might be stressed, with considerable validity, that even if the left-turning street-car train mentioned did take as much time as is required for twenty-one passenger automobiles to move straight through, the street-car train actually carried twice as many passengers as all twenty-one of the passenger cars.

The time taken to make turns is further influenced by the volume of pedestrian traffic moving parallel with the traffic stream from which the turn has been made. Such pedestrian data should be available when studies are being made in connection with the gains to be expected from prospective turn eliminations.

In making turn elimination studies, the alternate routes, with the interferences likely to develop, must receive careful consideration. It should be accepted as a fundamental principle in such studies that two alternate routes must be available for each turn eliminated: One to the motorist before he reaches the point where the turn is to be eliminated; the other, after he has reached this point, in case he has forgotten to take the first alternate route. As far as possible turn eliminations should follow some reasonably regular pattern. It is extremely disconcerting and it adds to the confusion, for a motorist, especially a stranger, to find turns eliminated at random. This, also, is a factor in reducing street capacity. Properly located and legible signs are essential. An origin and destination study of the traffic involved in the turn is also extremely helpful in determining a proper course of action.

The second type of delay is caused by marginal interference, which is due to: (*a*) Vehicles entering and leaving parking lots and garages; (*b*) vehicles parked along the curb; (*c*) vehicles pulling out from the curb and backing in; and (*d*) the natural slowing down of moving cars in the lane adjacent to a line of parked cars, etc. Loading and unloading at the curb has a similar effect in reducing street capacity to parked cars. This interference is more difficult to correct because facilities for off-street loading and unloading have not yet been provided in most cities.

The reduction in traffic capacity due to parked cars pulling into, and out from, the curb is naturally dependent on the degree of turn-over in the parked cars. This depends on the type of district, the parking regulation, and the degree of enforcement secured.

In down-town Pittsburgh the average stopping time of all cars parked or loading on the streets is 21.5 min; 48% are loading and unloading, requiring 8.5 min; 24% are parked while the occupant is shopping, requiring 24.5 min; 28% are parked for 40.5 min while the occupant has gone to his work or is engaged in recreation. On the 1-hr parking streets, the average parking time is 37 min.

The effect of some of these factors can be illustrated by examining the ratio of lane use of two typical Pittsburgh thoroughfares, shown in Table 6. The first of these thoroughfares is Sixth Avenue, a four-lane, two-way street with a line of street cars moving in each direction. The data were obtained at the near side of each intersection.

TABLE 6.—RATIO OF LANE USE, TYPICAL STREET

Block	Intersection	Lane No. 1	Lane No. 2	Lane No. 3	Lane No. 4	Remarks
	Sixth Avenue at:					
1	Fifth Avenue.........	0.077	0.923	0.762	0.238	Parking garage next block
2	Wylie Avenue........	0.499	0.501	0.715	0.285	No parking or stopping
3	Webster Avenue	0.499	0.501	0.715	0.285	No parking or stopping
4	Grant Street	0.506	0.494	0.650	0.350	No parking or stopping
5	Smithfield Street......	0.239	0.761	.....	.....	Partial parking
6	Wood Street	0.070	0.930	0.798	0.202	Considerable parking
7	Liberty Avenue.......	0.580	0.420	.....	.....	No parking or stopping
	Fifth Avenue at:					
	Grant Street.........	0.133	0.867	0.728	0.272	....
	Smithfield Street......	0.052	0.948	0.855	0.145	Parking in Lane No. 1
	Market Street........	0.237	0.763	0.687	0.313	....

At Fifth Avenue, which may be called the entrance to Block 1, 0.077 of the traffic was in the curb lane whereas 0.923 was in the adjacent lane. The reason is plain when it is known that in the next block there is a large parking garage with a capacity of about 600 cars. Motorists simply will not use the curb lane when they know they will be blocked by cars entering and leaving this parking garage. In other words the capacity of the street has been reduced nearly one-half. In the next three blocks, where there is no such interference and where the no-parking regulation is strictly enforced, there is a practically even distribution of traffic; in fact, in the third block the curb lane even carries slightly more traffic than the middle lane. The character of the business in the next block (Block 5, Table 6) is such that a certain amount of parking, or at least short-time stopping is permitted. The curb-lane use drops to 0.239. In the next block (Block 6) even more parking and stopping is permitted, and the curb-lane use drops to 0.070. The next block (Block 7) is a short one with parking again absolutely banned; the curb-lane use increases to 0.580. In the latter block, however, some influence is felt from the fact that considerable of the traffic turns to the right. As far as the law is concerned, a no-parking regulation is "on the books" for this entire side of Sixth Avenue, but the business use and degree of enforcement are different.

The second street that will be examined—Fifth Avenue—is one of the main streets in the down-town district. The parking ordinance calls for "no parking at any time" for all the blocks in question, on both sides of the street. In spite of this regulation, the fact remains that considerably more parking is permitted on one side than the other. On the north side, where this parking exists, the ratio of curb-lane use is 0.052 whereas, on the other side, it is 0.272.

Some factual data regarding the actual delays to traffic in the adjacent lane caused by cars pulling into, and out from, the curb have been obtained. Each car which pulls out from the curb interrupts the adjacent lane for an average of 11.25 sec, whereas each car backing into a parking space interrupts the adjacent lane 16 sec. The total number of observations for cars leaving as well as entering was sixteen. The more frequent the turn-over the more serious is this effect in reducing street capacity. It must be remembered, however, that since the time necessarily allowed for cross-traffic to move has already had its effect in reducing capacity, some time is available between blocks for delays such as these, without their complete effect showing on the over-all capacity of the street. There is also a speed-reducing effect on the lane immediately adjacent to the parking lane. Its capacity is reduced as compared to a lane not immediately adjacent to a parking lane.

The third type of delay is caused by internal interference, such as the weaving of cars from one lane to another, the conflicts that occur when cars pass each other in adjacent lanes when the lanes themselves are too narrow, the effect of vehicles of different characteristics, etc.

Formerly, the width of roadways was determined by the old three-fifths rule. This rule determined the width as three-fifths of the distance between property lines, which distance itself had been somewhat arbitrarily determined —usually being some multiple of ten. Thus, for a 30-ft street, the roadway was 18 ft wide; for a 40-ft street, 22 ft, etc. Such widths permitted no similarity in the actual lane widths available for moving traffic. In few cities has any one attempted to make the actual lane widths available for traffic movement, suitable for such movement, by adjusting the sidewalk width. Furthermore, in the establishment of parking regulations scant consideration has been given to the width of the roadway, so as to provide for a given number of lanes of adequate width, for moving traffic. In some cases an 8-ft lane is available, in others, 9 ft, and, in others, 10 ft. A lane that is too wide is economically wasteful from the standpoint of pavement cost; a lane that is too narrow inevitably decreases the capacity of the street.

The traffic flow on a city street is not directly proportional to the number of lanes. Arthur N. Johnson,[8] M. Am. Soc. C. E., has demonstrated that, for rural highways, three lanes had twice the capacity, and four lanes, three times the capacity, of two-lane roads, etc. The same effect is not so evident on city streets. In down-town Pittsburgh there are a number of four-lane streets that are actually carrying more traffic than other six-lane streets with about the same apparent congestion. The difference is in the interference factors, which are variable from street to street.

Rough pavements limit street capacity, as do steep grades. Grades of less than 5% have little effect on modern traffic, however. Poor street illumination has its effect on traffic speeds, with a consequent effect in reducing capacity in districts where street use is heavy after dark.

Where routings are such that traffic must weave from one lane to another in order to move into the proper position for a turn, another factor which reduces traffic capacity is introduced, especially where the weaving must take place across several lanes. Some brief studies on this subject, applicable to down-town Pittsburgh conditions, were made on one down-town street (Second Avenue). These studies cover the weaving from one lane to a second lane (not multiple-lane weaving across several lanes). The studies were made in the evening peak period.

The method of the study may be briefly stated as follows: A measured course was laid out on Second Avenue, between Smithfield and Grant Streets, on the side for eastbound traffic. Several automobiles were hired, and drivers were instructed to weave from the lane nearest the center of the street to the next lane to the right, starting always at the first line of the measured stretch. The distance in which the weave was made and the speeds of the weaving vehicle were recorded. The speeds of following vehicles on both lanes were also recorded, as well as the speed of traffic when there was no weaving. Other observers checked the volume of traffic per lane in given time periods, so that the density of the traffic through which the test automobiles did their weaving was determined (see Table 7).

The composite average of these data shows a reduction of approximately 40% in the speed of traffic in the lane from which the weaving started, and a reduction of more than 30% in the speed of traffic in the lane to which the weaving vehicle moved. The average delay to vehicles in the lane from which the weaving started was 0.75 sec, and 2.6 sec in the lane to which the weaving vehicle moved. The range of weaving delays was from a negligible time to 9 sec. Translated into terms of effective headway, each weaving vehicle displaced from one to six vehicles moving straight through, the over-all reduction in capacity of the street depending on the number of weaving vehicles. Weaving, therefore, is another interference factor to be avoided if a street is to attain its maximum capacity.

Other elements that reduce the effective width of traffic lanes are structures such as loading zones and safety islands for street cars. Wherever possible the width taken out of traffic lanes by such structures should be compensated by cutting back the sidewalk opposite them a similar or greater distance. Even so, the necessary diversion of the traffic from a straight line is certain to reduce the over-all capacity of the street. Pillars of elevated railways are similar obstructions for which compensation should be made. On the other hand, structures such as loading zones for street cars introduce a helpful factor in that they permit traffic to move past street cars which are stopped to take on passengers.

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[8] *Proceedings*, Highway Research Board, 1930, pp. 218–224.

TABLE 7.—DELAYS DUE TO WEAVING FROM LANE NO. 3 TO LANE NO. 2*

Period No.:	Fifteen-minute period (afternoon), ending:	SPEED, IN MILES PER HOUR					Average distance required, in feet	TRAFFIC FLOW (NUMBER OF VEHICLES)				DELAY, IN SECONDS, CAUSED TO TRAFFIC BY WEAVING VEHICLES		Time, in seconds, required to weave
		With no weaving in:		With weaving in:		Of the weaving vehicle		Actual flow in vehicles per 15-minute period		Equivalent flow, in vehicles per hour				
		Lane No. 3	Lane No. 2	Lane No. 3	Lane No. 2			Lane No. 3	Lane No. 2	Lane No. 3	Lane No. 2	Lane No. 3	Lane No. 2	
	(1)	(2)	(3)	(4)	(5)	(9)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
						(*a*) FRIDAY, AUGUST 9, 1929								
1	4:30	17.7	17.3	6.2	...	9.3	65	115	142	460	568	9.0	...	4.75
2	4:45	17.2	17.3	11.7	10.2	7.0	52	107	162	428	648	2.3	3.5	5.07
3	5:00	18.0	16.1	...	...	...	...	177	115	708	460	...	...	2.2
4	5:15	14.4	13.7	9.9	6.8	6.9	55	171	215	684	860	2.7	6.3	5.4
5	5:30	11.7	11.8	8.1	10.3	10.3	65	199	205	796	820	3.2	1.1	4.3
6	5:45	14.2	13.5	...	13.7	6.1	73.5	149	165	596	660	...	1.9	8.2
Average.....	.....	15.8	14.7	8.7	9.9	7.4	60	153	167	696	700	3.8	2.5	4.8
						(*b*) FRIDAY, AUGUST 16, 1929								
7	4:30	19.0	14.6	12.2	8.8	10.2	74	117	169	468	676	2.8	3.8	4.9
8	4:45	15.6	13.6	14.1	10.9	8.8	66	158	156	632	624	0.8	1.0	5.4
9	5:00	16.2	12.8	10.2	19.2	6.4	82	150	186	600	744	3.0	2.6	8.4
10	5:15	17.7	14.3	...	12.0	6.0	80	136	227	544	908	...	2.2	9.0
11	5:30	13.1	12.4	...	9.0	9.0	58	199	272	796	1 088	...	2.6	4.1
12	5:45	15.1	14.1	9.5	12.8	7.0	75	168	221	672	884	3.2	1.0	7.2
13	6:00	17.6	14.7	10.7	13.1	9.8	90	151	155	604	620	3.1	0.8	6.3
Average		16.8	14.7	10.9	10.1	8.3	70	153	198	644	906	2.6	2.2	6.0
Composite average..		16.1	14.7	9.8	10.2	8.0	66.4	153	184	696	906	3.4	2.6	5.6

* Lane No. 3 is next to the middle of the street.

Another factor in internal interference is based on the character of the vehicles themselves. Taxicabs are a delaying factor, not so much because the vehicles are incapable of moving with the traffic stream, but because the drivers lag in their effort to pick up passengers, or to get into desirable taxi stands. If cruising is prevalent, the extra volume of traffic added to the street displaces vehicles of other types. For fifteen intersections selected at random in downtown Pittsburgh, taxicabs comprised more than 8.5% of the total motor-vehicle traffic and in one block (Fifth Avenue, between Smithfield and Wood Streets) taxicabs comprised 25.8% of the total motor-vehicle traffic. In four other blocks they exceeded 10% of the total. Of the 4 591 cabs counted in this study, 76.7% were empty.

In another study (Table 8) covering 856 cabs and three intersections, 84% of the cabs were empty. The first appearance of a cruising taxicab was considered legitimate and, therefore, this vehicle was not included in Column (6), which includes only the second and subsequent appearance of each cab. During a 30-min period cabs were observed to enter an intersection two or more times as follows:

TABLE 8.—STUDY OF TAXICAB CRUISING

Time of observation	Location	No. of observations	Percentage of Observations:		
			Loaded	Empty	Cruising
(1)	(2)	(3)	(4)	(5)	(6)
8:00 A.M. to 10:00 A.M.	William Penn Way and Oliver Avenue	335	16	84	23
4:00 P.M. to 6:00 P.M.	Grant Street and Diamond Street	340	19	81	31
February 8, 20, and 27, 1933.	Fifth Avenue and Wood Street	181	11	89	25

No. of appearances	No. of cabs	No. of appearances	No. of cabs
2	80	6	2
3	32	7	1
4	6	9	1
5	6		

Even granting that the first appearance of an empty cab was legitimate on the assumption that it was going to a call or a stand, 25% of the empty cabs were cruising; that is, they appeared, empty, two or more times at the same location within 30 min. Taxicab stands themselves furnish the same type of interference as parked cars. Such stands, of course, should never be permitted, except on streets where other vehicles are permitted to park.

Buses are also a delaying influence, in that they usually stop at the curb to pick up passengers and as a consequence delay the following traffic. One item receiving considerable attention is whether the bus stop should be on the near side, or on the far side of the intersection. Practice varies in different cities, although the prevailing practice seems to be to have the stop on the near side of the intersection so that stops necessary on account of the red light can be utilized for picking up passengers. On the other hand, when the bus arrives on the green light, such stops delay following traffic, which otherwise would go on through. Physical characteristics, such as the width of the street, number of lanes for moving traffic, whether parking is permitted or not, whether the street is flowing nearly to capacity, the number of right turns made into the street, etc., are determining factors in any individual case, although the practice must be made uniform throughout the congested district.

The effect of trucks and other commercial vehicles has been indicated previously. Due to their slower speeds they reduce the capacity of any thoroughfare. For instance, a traffic load of one-third trucks reduces the capacity about 20% as compared to a flow consisting entirely of passenger cars. Wherever possible, trucks should be segregated on certain streets. Trucks following regular routes to established terminals should be carefully considered and their route established so as to cause the least possible interference.

Because of their fixed use of the street-car lane, street cars limit following traffic in congested districts to the speed that these vehicles themselves are able to maintain, and thus lower street capacity. Many cities are using antiquated types of cars with not only slow speeds, but also slow accelerations.

The modern, high-speed street car developed by the President's Conference of the American Transit Association has an acceleration and speed enabling it to hold its own with motor traffic in the traffic stream, with the exception of the necessary stops to pick up and discharge passengers. This new car is being introduced in a number of cities. Of necessity, when the car stops to take on passengers, it blocks its own lane. Careful signal planning and study of the

TABLE 9.—LOADING TIME OF STREET CARS, ON "RED" AND "GREEN" TRAFFIC SIGNALS (OBSERVATIONS TAKEN BETWEEN 4:30 P.M., AND 6:00 P.M.; 104 SIGNAL CYCLES AT EACH LOCATION; TOTAL, 416 CYCLES)

Description	No Loading Platform			Loading Platform	Total
	Traffic from north on:				
	Wood Street at Fifth Avenue	Sixth Street at Penn Avenue	Grant Street at Fifth Avenue	Traffic from west on Grant Street at Sixth Avenue	
(1)	(2)	(3)	(4)	(5)	(6)
Total number of observations	87	60	78	56	281
Percentage of total time used for loading	23.6	21.9	14.9	16.8	19.2
Percentage of total time that is stop time	55.8	48.0	46.1	48.0	49.5
Percentage of Time That Cars Are Loading Passengers on:					
"Stop" time	34.7	27.2	14.7	20.0	24.6
"Go" time	9.4	17.0	15.0	13.9	14.0
Percentage of Time During a Cycle, in Which a Car is Loading Passengers on:					
"Stop" time	74.0	44.2	41.3	46.1	51.4
"Go" time	43.3	26.9	50.0	33.6	38.4
Percentage of Time That Cars Are Loading Passengers Exclusively on:					
"Stop" time	47.2	43.4	28.2	35.7	38.8
"Go" time	0.0	5.0	44.9	12.5	16.0
Percentage of Loading Time That Occurs on:					
"Stop" time	82.3	59.8	45.5	57.2	63.3
"Go" time	17.7	40.2	54.5	42.8	36.7
Percentage of Cars That Arrive on:					
"Stop" time	56.3	95.0	50.0	60.7	63.7
"Go" time	43.7	5.0	50.0	39.3	36.3
Percentage of Time, During a Cycle in Which Cars Are Loaded, That Is, Actually Used for Loading:					
"Stop" time	21.6	56.3	30.1	41.2	36.5
"Go" time	47.0	61.5	35.4	43.3	47.0
Ratio of the Loading Time on "Stop" Signals to Loading Time on "Go" Signals	4.64	1.49	0.84	1.33	1.73

characteristics of individual stops will enable the stopping and loading of street cars on the "red" signal, to a surprisingly large degree, thus eliminating much of this delay. Loading zones permitting the by-passing of motor traffic are a further help.

Some observations of results actually obtained in this respect at four rather heavy loading points selected at random in down-town Pittsburgh are presented in Table 9. At two of these points, loading platforms which permit the by-passing of other traffic, have been installed; at the other two, all automobile traffic must stop while the street car is loading. The readings were made during the evening peak from 4:30 P.M. to 6:00 P.M., and a total of 281 street cars was observed. The average "stop" time for the four points was 49.5% of the total time.

These data show that (see Column (6), Table 9): (1) 63.3% of the total loading time occurred on "stop" time; (2) 38.8% of the cars loaded exclusively on the "stop" time, and only 16.0% loaded exclusively on the "go" time; (3) some loading on the "go" time occurred in only 38.4% of the signal cycles; (4) 36.5% of the "go" time in the cycles in which cars loaded, was actually used for loading, and 47% of the "stop" time in the cycles in which cars loaded, was actually used for loading; and (5) 1.73 times as many seconds were used in loading on the "stop" time as on the "go" time.

A distinct difference was observed between the locations where loading platforms were installed, and those where patrons were required to board street cars without this protection. This difference illustrates not only the value of the loading platforms themselves, but the fact that the design of the signal system had actually taken this difference in street-car loading procedure into consideration. At the two locations without loading platforms (Columns (2) and (3), Table 9), the percentage of total loading time occurring on "stop" was 82.3 and 59.8 respectively; at the two locations with such platforms (Columns (4) and (5), Table 9) the corresponding percentages were 45.5 and 57.2, because it was not so important to have cars load on "stop" in the latter case. At the locations without the platforms, street cars loading exclusively on the green light were zero and 5%; at the locations with platforms these values were 44.9 and 12.5 per cent. Furthermore, the ratio of the seconds of loading on "stop," to the seconds of loading on "go," was 4.64 and 1.49 where there were no loading platforms, but 0.84 and 1.33 where there were loading platforms.

In discussing the effect of taxicabs on street capacity, the writer mentioned the fact that, in so far as considerable of this traffic was probably not serving a useful purpose, it was occupying the space that could be occupied by other vehicles, thereby reducing the street capacity to useful traffic.

In most congested districts there is another type of traffic, present in even greater volume, that is reducing street capacity in the same manner. This is through traffic, as contrasted to the strictly local, business-producing traffic.

In down-town Pittsburgh 49.7% of the vehicles entering between the hours of 6:00 A.M. and 12:00 midnight remained in the district less than 20 min and left by a different route from that by which they entered. The volume of this through traffic is illustrated by Fig. 3. In the evening peak period from 4:00 P.M. to 6:00 P.M., it amounted to 53.3 per cent. Progress through restriction

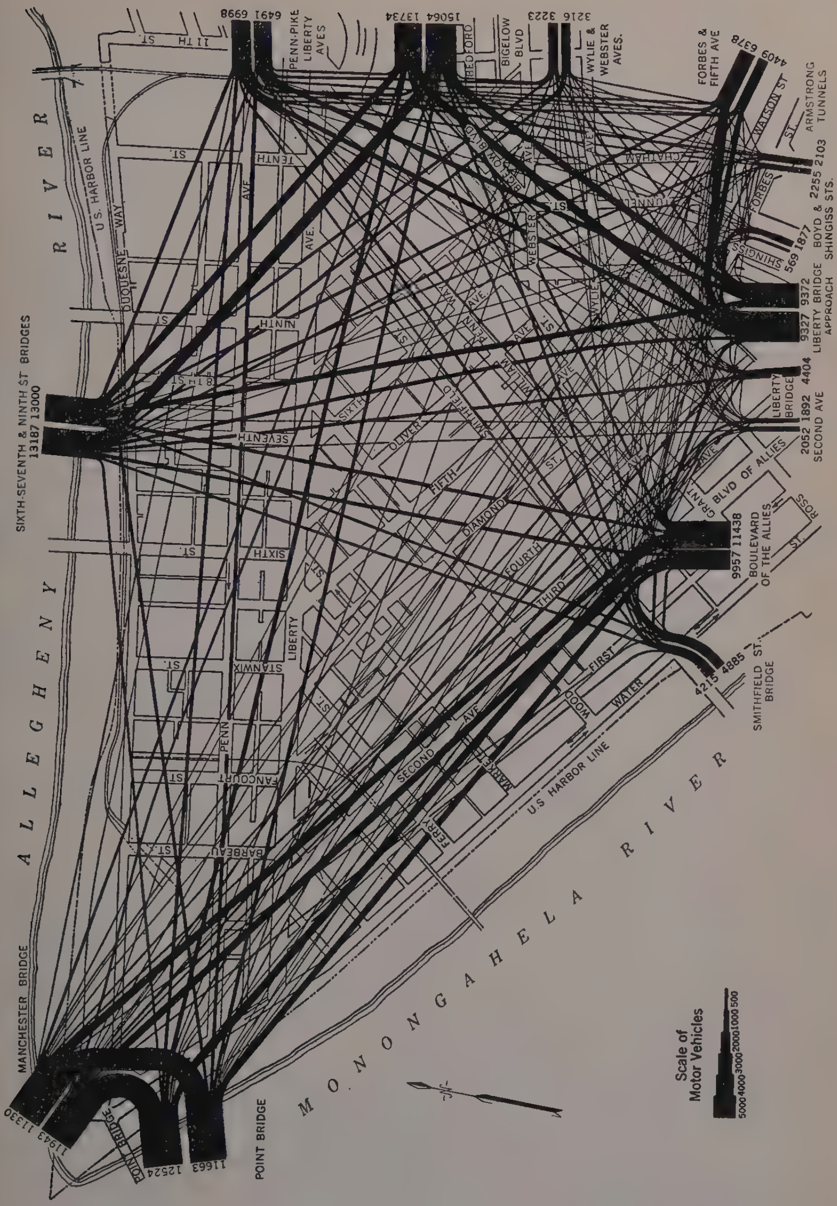

FIG. 3.—THROUGH TRAFFIC FLOW, CENTRAL BUSINESS DISTRICT, PITTSBURGH, PA.

of parking, elimination of turns, the creation of one-way streets, more efficient traffic signal systems, etc., cannot be continued indefinitely.

Traffic that merely passes through congested districts on its journey from one outlying district to another, adds nothing to the prosperity of a congested area. The tremendous relief from congestion which would result from the elimination of 40% to 50% of the present traffic is evident. What a rejuvenation business organizations could experience through the removal of that 40% to 50% of non-business producing traffic and its replacement by an equal volume of business-producing traffic! Such a change would not entail any greater burden on the streets of Pittsburgh than at present, and could be accomplished by the construction of a close in by-pass system around the congested district. It is essential if permanent traffic relief is to be secured. In the meantime some improvement could be accomplished by establishing, and thoroughly marking, preferred through routes over existing streets. Normally, through traffic filters to a considerable extent from one street to another in down-town areas, producing unnecessary confusion. A preferred route will often shunt this traffic flow to an adequate street hitherto neglected.

The fourth type of delay is due to medial or center-line interference. This is the interference that occurs between traffic moving in opposite directions. On radial, by-pass, and distribution streets, as well as on the open highway, where such highways approach large centers of population, it can best be eliminated by the use of a divided roadway with a center strip 15 to 30 ft wide separating the roadways used by traffic moving in opposite directions. Such a construction is largely impractical in congested districts. It is fortunate, therefore, that in such districts this type of interference is at a minimum and may be neglected, in view of the large amount of interference of other types.

Conclusions

On the basis of data presented herein, the following conclusions can be advanced:

(1) The traffic capacity of a single lane of a street in a congested city district, with interferences eliminated, is from 2 000 to 2 500 passenger automobiles per hr.

(2) Interference by cross-traffic at grade inevitably reduces the capacity of any street approximately 50 per cent.

(3) The stopping of a lane of traffic reduces the capacity of that lane 35% to 50% when the stops occur close together.

(4) In the absence of pedestrian control, as much as 23% of the pedestrian traffic crossing the street will interfere with vehicles, thus reducing the street capacity. As many as 56% of the pedestrians crossing the street in congested districts cross against the traffic. Crossing in the middle of the block or "jaywalking" is also prevalent. These practices, which reduce street capacity, can be greatly reduced with adequate pedestrian control.

(5) On the average, a passenger automobile turning left may delay following traffic a length of time equal to that required for four such vehicles to go straight through; a street car and trailer turning left may delay following traffic a length

of time equal to that required for twenty-one passenger automobiles to go straight through.

(6) A large parking lot or garage with the entrance on a main thoroughfare may have the effect of reducing the capacity of that street from 20% to 40% of the traffic which would otherwise use it. The extent of the reduction depends on the number of lanes of moving traffic on the street.

(7) The effect of curb parking is to reduce the traffic capacity of the street in question to 50% of the traffic that could otherwise use it.

(8) The traffic flow of city streets is not proportional to the number of lanes. City streets in congested districts do not follow the same rules in this regard as rural highways because of considerable variation in interference factors from street to street.

(9) Where street-car loading platforms, safety islands, elevated railway pillars, etc., obstruct the roadway, the curb should be "cut back" a distance at least equal to the width taken out of the roadway by such structures, to avoid any reduction in lane widths.

(10) One automobile weaving from one lane to another causes a change in effective headway equivalent to six vehicles moving straight through.

(11) Empty taxicabs, to a considerable extent cruising, may comprise as much as 20% of the motor-vehicle traffic on a street. To the extent that they are not serving a useful purpose and are occupying the space that could be occupied by other motor vehicles, they are reducing street capacity.

(12) Trucks reduce the capacity of a street 25% to 40% as compared to passenger automobiles. With trucks equal to approximately one-third of the traffic, the reduction in capacity is approximately 20% as compared to all passenger automobiles.

(13) Antiquated types of street cars reduce street capacity through slow speeds and slow accelerations. Modern street cars are available which can hold their own in the traffic stream. Street cars delay following traffic when they stop to take on passengers. Through the use of loading platforms and careful traffic signal system design considerable of this apparent reduction in capacity can be eliminated.

(14) As much as 50% of the traffic in congested districts is non-business producing, through traffic. This must be largely eliminated by the construction of convenient by-passes, if permanent traffic relief is to be secured. Improvement in this direction can usually be accomplished by establishing and thoroughly marking preferred through routes over existing streets.

ACKNOWLEDGMENT

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REMEDIES TO RELIEVE TRAFFIC CONCENTRATION

By Donald M. McNeil,[9] Jun. Am. Soc. C. E.

Synopsis

Growth of the automobile has resulted in congestion in business districts where the street system has not been expanded to compensate for this growth. Traffic interference, both pedestrian and vehicular, causing delays is discussed, and remedies are suggested. From an engineering standpoint most delays can be eliminated, but all improvements should be weighed from the standpoint of cost *versus* benefit to business. Inexpensive suggestions are given in the paper for relieving cross, marginal, internal, and medial interference.

Introduction

With rapid development of the automobile, practically every city has been confronted with serious congestion in its business section. This congestion, which can be defined as an over-crowded condition causing a decided reduction in the rate of speed of the movement of traffic, unquestionably reflects on the volume of business in the area. The development of the automobile has been so rapid that municipalities have been unable to devise regulations and develop improvements fast enough to cope with the situation. Re-arrangement of the existing street systems is decidedly expensive and, in many cases, from a financial and economic standpoint it is impossible; and the engineer is faced with the problem of examining the existing street systems to determine whether the ultimate capacity is now being obtained, and if not, to develop methods for obtaining greater capacity.

When the area of the main business section of Pittsburgh, Pa., is considered as 0.54 sq mile, comprising 14.9 miles of streets, on which, at present (1937), 75 000 vehicles enter daily; and when one considers the necessity of moving 19 127 vehicles, plus 541 street cars, in the peak hours, during which period approximately 100 000 persons use the various modes of transportation, a series of delays becomes extremely costly. The time required for automobiles to depart from the down-town area is five times greater, and, for street cars, three times greater, than on a thoroughfare on which there are no delays. The present average time required to depart from the down-town area is approximately 20 min. This average is three times greater than that which would be required on a street system with no delays. Reducing this to man-hour delays at the current rate of day labor (40 cts per hr), the cost of delays in Pittsburgh during only the peak hours of one day equals $10 000, not to mention the cost of the accidents which always accompany such delays. Therefore, it is easy to appreciate the exorbitant loss with which each municipality is faced under present street systems. The analysis for the peak period in Pittsburgh is more or less general in all cities.

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It would be unreasonable to expect a street system improved so as to avoid all delays, because of the tremendous cost involved. However, many of the delays that are now causing congestion in cities of the United States can be eliminated, and the capacity of their street systems can be increased; but it is essential that, in each case, the benefits to be gained shall be weighed carefully against the cost of such improvements (which improvements can be decidedly expensive), and careful decision must be made as to which solution is warranted.

Where the expression, "speed," is used in this paper, it must be kept in mind that the prevailing average gross speed in a business section varies between approximately 5 and 10 miles per hr. From studies made on the relation between the capacity of a lane and speed, it was shown that capacity varied approximately in direct proportion to a speed of about 23 miles per hr, but for greater speeds, the capacity decreased as the speed increased. This condition is due to the difference in headway because of the distance required to stop a vehicle in case of an emergency. Therefore, when reference is made to speed in this paper, unless otherwise noted, it relates to a value of less than 23 miles per hr—that critical speed of the volume graph. The various delays possible may be classified in four groups as follows:

(1) Cross-Interference: That interference to a stream of traffic moving in one direction crossing the path of another stream of traffic on the same plane.

(2) Marginal Interference: That type of interference which occurs between moving vehicles and other vehicles, persons, or objects alongside the outer edge of the roadway.

(3) Internal Interference: That type of interference which occurs between traffic units moving in the same direction in the same traffic area.

(4) Medial Interference: That interference between vehicles moving in opposite directions at, or near, the center of the roadway.

Remedies for Delays Due to Cross-Interference

To minimize cross-interference, the necessity of traffic in one direction to cross-traffic in another direction must either be eliminated, reduced, or regulated so that the traffic involved does not arrive at its point of conflict at the same time. In cross-interference, the delay encountered is the time lost due to the necessity of slowing down and, in many cases, stopping, and re-starting. At an intersection it can be eliminated completely by the construction of grade separations on the intersecting streets or, at a series of consecutive intersections, by an elevated roadway through that section of the town. Such grade separations are decidedly expensive, frequently requiring the acquisition of more land area. Theoretically, the average typical intersection reduces the capacity of the intersecting streets to 50%, not considering the delay due to slowing down, stopping, and re-starting, because when traffic is moving on one street, the traffic on the other street must wait.

The grade-separation intersection does not require a change in the speed of traffic traveling straight through it. However, a slightly reduced speed is required by that traffic traveling from one street to the other street, provided the streets have ramp connections. Such ramp connections should be designed so that drivers wishing to travel from one street to the other may do so by a right turn. This type of movement provides less delay to traffic using the street on which the vehicle is traveling. The ramps that connect the roadway to which the vehicle is traveling should preferably be designed so that only a right turn can be made. The "clover leaf" design is an example.

Cross-interference can be greatly reduced by signal control or other automatic devices, so that cross-traffic does not arrive at points of conflict at the same time. This is illustrated by the results of the flexible progressive traffic signal system installed in Pittsburgh in 1928. Prior to this time the streets of the city were seriously congested. Since the installation of the system (in which time there has been practically no physical change in the street system) the traffic volume has increased approximately 40%; yet the average speed of traffic in this section was practically the same in 1936 as it was in 1928. This success is due largely to the frequent modification of the signal timings at individual intersections and the co-ordination pattern that has been developed to provide progressive movement.

A study in 1934 of the action of approximately 1 000 000 pedestrians revealed the fact that 56% of the total number approached the intersection on the red signal, and one-half, or 28%, showed no regard for it. Of the pedestrians who disregarded the red signal, 22.9% interfered with traffic that had the right of way on the street the pedestrian was crossing. These data show that a pedestrian interferes with the movement of street traffic on an average of one out of every fifteen times he crosses an intersection. This value was obtained on fixed time signals having a signal cycle of 50 sec. Further studies were made at these same intersections when the signal cycle was 60 sec, and the results were practically the same as those reported when the signals were operating at a 50-sec cycle. Delays due to this interference by pedestrians can be eliminated only by rigid enforcement, requiring the strict observance of the law prohibiting pedestrian movement during the red-signal period. In determining the proper length of the signal cycle, consideration should be given to the length of time that pedestrians are required to wait to cross the street. Studies made by Mr. J. Rowland Bibbins[10] indicate that the total violations of pedestrians crossing against traffic on a 20-sec red period would be perhaps only one-half that on a 30-sec red period, and the latter only about one-third that on a 60-sec red period. Although no data are available of the number of pedestrians crossing in the middle of the block, thus interfering with the movement of traffic, it is evident that delays from this cause are considerable. Records indicate that this jaywalking of pedestrians is decidedly dangerous and frequently causes accidents. The remedy for this type of delay is a rigid enforcement of the laws eliminating jaywalking and confining pedestrian crossings to the intersections.

When turning right or left at an intersection, a vehicle is required to cross through the pedestrian traffic crossing on the cross-walk of the throat of the street into which the vehicle is turning. No data are available to show the number of times such vehicles interfere with pedestrians. However, from studies of turn delays made in Pittsburgh at typical business intersections, it was found that each right-turning vehicle delayed through traffic, traveling in the lane from which the vehicle turned, an average of 0.7 sec; and that each left-turning vehicle delayed not only through traffic in the lane from which the vehicle turned, an average of 1.7 sec, but also delayed the traffic in the opposite direction an average of 2.6 sec. This excessive delay of 2.6 sec is due largely to the interference of pedestrians legally crossing the cross-walk of the throat into which the vehicle is turning. Some of the remedies for these turning delays are as follows:

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[10] *Transaction*, National Safety Council, Annual Safety Congress, 1930, p. 126.

(*a*) If there are many turning vehicles, in a heavy pedestrian traffic, it is possible that the remedy may be the provision of a pedestrian interval in the signal sequence, during which period no vehicles shall move.

(*b*) If it is possible to sacrifice the flexibility of the intersection, the remedy could be the restriction of all turns, or of certain turns.

(*c*) If property can be obtained reasonably, it is possible that the remedy would be the construction of a circular arrangement, so that traffic desiring to turn left would proceed slightly beyond the intersection, turning right around a circle and waiting for the red light before completing the movement through the intersection. Such an arrangement would likewise interfere materially where there was heavy pedestrian movement, and would require the handling of the traffic twice through the intersection.

(*d*) It is possible that the remedy would be the construction of a circular roadway requiring all traffic to move in a rotary manner. This circle should be of ample diameter, preferably 250 ft, or more. Although it would eliminate cross-interference, it would create a weaving interference—to be described subsequently.

(*e*) Possibly, at least a partial remedy would be to construct connecting tunnels between basements of large stores located at the intersections.

(*f*) A possible remedy would be the provision of pedestrian under-passes where traffic would not permit turn eliminations, or the provision of a pedestrian interval. The experience with pedestrian under-passes on outlying main arteries in Pittsburgh has been extremely unsatisfactory, because of their little use. If pedestrian under-passes are provided, they should be extremely sanitary, well-lighted, designed to require the least possible exertion on the part of pedestrians walking through them, and preferably arranged to attract their use, such as with show windows and connections to buildings. Pedestrians should be educated as to their advantage.

In summarizing, the remedies for delays encountered by cross-interference are likely to be found in one or more of the following:

I.—Elimination of cross-interference due to pedestrians:

- (*A*) Rigid enforcement of pedestrian regulations: (1) No crossing on the red signal light; (2) no jaywalking.
- (*B*) Provision of a pedestrian interval, during which period no vehicles shall move, and on the other periods, no pedestrians shall move.

- (*C*) Connecting the basements of large buildings by pedestrian tunnels. (Partial remedy.)
- (*D*) Construction of pedestrian under-pass tunnels.

II.—Elimination of cross-interference due to vehicles:

- (*A*) Provision of grade separation intersections or elevated roadways.
- (*B*) Provision of a progressive co-ordinated signal system, or its equivalent.
- (*C*) Elimination of turning by vehicles.
- (*D*) Provision of a far-side, right-turn, circle roadway to replace left-turning movement.
- (*E*) Provision of a circular roadway.
- (*F*) "Channelize" wide uneven intersections.

Remedies for Delays Due to Marginal Interference

Marginal interference is usually created by the granting of conveniences to a few at the expense of a majority. Its remedy in general is inexpensive and involves limiting these conveniences. The greater part of marginal interference is caused by street and off-street parking, and the loading and unloading of vehicles. Its entire elimination is impossible, as the Courts have ruled that a municipality cannot restrict a vehicle from stopping for that period of time necessary to load or unload a vehicle expeditiously. It is an admitted fact, substantiated by the Courts, that streets are primarily for the purpose of the moving vehicle. However, such a vehicle is decidedly limited if it cannot stop to discharge any of its occupants or materials. The necessity of permitting vehicles to stop for loading or unloading has resulted in a considerable restriction of the use of the curb lane. From lane-ratio data available, the curb-lane usage of a street on which there is a moderate loading or unloading traffic is less than one-half that of the center lane, whereas on other streets where there is considerable loading, the curb-lane usage becomes practically zero. Such delays due to loading and unloading can be eliminated by restricting all stopping of vehicles (at least during the rush periods), in which case, unless other interference is encountered the lane distribution ratio will be approximately equal, as indicated by other available data. This elimination of stopping has the possibility of providing an increase of 40% in the capacity of a four-lane street. Where parking is permitted on a street, it is found that there is generally no usage obtained from the curb lane. If this parking were prohibited, and if reasonable loading were permitted in the curb lane, it would be possible in some cases to increase the capacity of the street 50%; and, with the restriction of loading during the peak periods, it would be possible to increase the capacity of the street 100 per cent. This statement, of course, is made with the provision that the curb lane has ample width for moving vehicles and that the intersections are capable of handling this additional traffic.

In addition to the obstruction of the curb lane, the parked or loading vehicle interferes with free-moving traffic in the second lane by pulling in and out of the curb lane. The seriousness of this delay depends upon the number of vehicles loading and unloading, and the rate of turn-over of parked cars. It has been observed, particularly in the business section, that the traffic in the lane adjacent to parked vehicles proceeds at a lower rate of speed than it would have traveled had there been no parked vehicles present. This is due to the preparedness of the operator to be able to stop his vehicle in the event that a parked car suddenly pulls out from the curb lane, or a jaywalking pedestrian steps from between parked cars into the path of his vehicle. If parking were eliminated, there would be no cars to pull out from the curb lane, and the distance in advance that the operator would be able to see the jaywalking pedestrian would permit the vehicle to progress at a higher rate of speed. In January, 1928, the City of Chicago, Ill., put into effect a blanket "No Parking" ordinance in the Loop District. In comparing the data of 1926 with that of several years subsequent to the restriction of parking, it is found that, in spite of 18.3% increase in traffic, street-car speeds increased from 18% to 33%, and speeds of all traffic increased 15% to 40%; in addition, total accidents were reduced 10 per cent.

The delays caused by vehicles entering and leaving parking lots and garages is another form of marginal interference. Off-street parking facilities are essential in a business section, and as it becomes necessary to restrict more and more street parking, the off-street parking facilities will be increased. Their success depends largely upon their location and their entrances, and it is for this reason that proprietors endeavor to locate them centrally and with entrances on main thoroughfares. The location of entrances to parking lots or garages on a main artery is similar to the addition of an intersecting street and, unfortunately, these lots and garages receive and discharge their tenants during the peak periods of traffic movement. Efforts should be made to have entrances to parking lots and garages open on to streets having the least congestion, and preferably on to streets that do not have through-artery characteristics. These lots and garages should be designed with sufficient reservoir space and operated with a sufficient number of attendants, so that vehicles desiring to enter them will not be required to wait on the street or the sidewalk before admittance. The exits should be located so that vehicles can be discharged on to a secondary street and, by way of connecting streets, distribute themselves to their respective main arteries.

Remedies for Delays Due to Internal Interference

Internal interference, being the type that occurs between traffic units moving in the same direction in the same traffic area, is due in general to the difference in speeds of the various vehicles, the tendency of certain vehicles to foul the lane or lanes adjacent to that in which they are traveling, and any physical condition of the roadway that may change the speed at which the vehicle is traveling.

Many of the streets in the business sections were planned during those periods when traffic consisted largely of horse-drawn wagons, bicycles, and other slow moving vehicles, with the result that in many cases the lane widths are not adequate for the free movement of present-day traffic. Formerly, it was the practice in designing streets, to make the roadway itself three-fifths of the street width; thus, when the street was 30 ft wide, the roadway was made 18 ft wide; for a 40-ft street the roadway was made 24 ft wide, etc. From an analysis of such roadway widths it will be noted that there is no similarity in the actual lane widths for moving traffic. The current design of new roadways in a business section provides lane widths for moving traffic of 9 and 10 ft, preferably 10 ft, and sufficient sidewalk area for the future development of that section. Where the lane widths are not of proper width (at least 9 ft) the speed of traffic is reduced materially, and thus the capacity of the street is curtailed.

Pedestrian traffic has increased materially in business sections, and in many cases sidewalks are not adequate to accommodate pedestrians properly. Some cities have attempted to correct the roadway deficiency by encroaching on the sidewalk areas, with the result that in many cases pedestrian traffic has been penalized. One possible remedy for an over-crowded condition of both vehicles and pedestrians would be to elevate the sidewalks to provide more roadway area. This type of improvement, while decidedly less expensive than the construction of arcades in existing buildings, or the razing of buildings, places an inconvenience on pedestrian traffic, unless the plan is adopted for an entire area, as it requires the constant walking up and down to, or from, the upper sidewalk.

Many of the old street designs provided for such small curb radii at the intersections as 6, 8, and 10 ft, which required traffic that was turning right from and on to relatively narrow streets, to proceed at a slow rate of speed. Frequently, such traffic is obliged to encroach on the wrong side of the roadway in making the turn. In remedying this condition by enlarging the curb radii, the engineer must be careful that the curb is not made so inviting as to encourage an excessive rate of speed, thus subjecting pedestrians to a hazard. Where pedestrian traffic is relatively light and the speed of traffic is approximately 25 miles per hr, curb radii of 35 ft are preferable. Where pedestrian traffic is heavy, it is suggested that the curb radii be modified to 20 ft, which will permit a comfortable speed for passenger cars of 10 miles per hr, and in some cases as much as 15 miles per hr, and a somewhat slower speed for commercial vehicles having large turning requirements. Where the street paving is in poor condition, or where it is decidedly slippery for driving, the speed of traffic is considerably slower than traffic traveling on improved non-skid pavements. Street car rails also have a tendency to reduce the speed of traffic, because of the possibility of skidding during slippery periods. At night, the speed of traffic in busy areas that are congested, is somewhat governed by the degree of visibility afforded the driver. Where there is sufficient street lighting, whether by store windows or street lights, or a combination of both, so that the driver can readily see any object that may appear suddenly in his path, he is inclined to drive at a faster speed than at places where he must rely largely on the headlights of his vehicle.

Another form of internal interference is that caused by vehicles weaving from one lane to another. In order to permit flexibility of a street system, this weaving is essential in many cases. Frequently, however, vehicles weave from one lane to another, due either to the inattention of the driver, or to his desire to travel faster than the vehicle preceding him. In many cases, this unnecessary weaving can be eliminated by "channelizing" the roadway (that is, by establishing closed lanes), and requiring persons traveling in a given lane to confine themselves to it. Frequently, the marking of the street will help materially to guide motorists. In a preceding part of this paper, a suggested solution for cross-interference is to provide a rotary circle of proper diameter. The success of this type of a roadway is due to the elimination of cross-interference at the sacrifice of weaving interference. The number of arteries connecting the rotary circle and the movement of the traffic using the circle determines its capacity. Unfortunately, sufficient data are not available to determine the exact capacity of a circle; nor are there sufficient developments of a rotary circle from which to obtain data, due to the fact that of the circles in use to-day, their traffic movements, roadway design, diameters, etc., are different. From such data as the writer has been able to obtain, a circle having an island 250 ft in diameter and a roadway 50 ft wide, has a capacity of approximately 5 000 vehicles per hr. This capacity is considerably less than that of a grade separation of the "clover leaf" design, but considerably less expensive.

The success of any business area depends upon the facilities available to the business houses to receive and deliver their merchandise, which often requires the use of large trucks. In general, these trucks travel at a decidedly slow rate of speed, compared to passenger automobiles and street cars, and require more lane area when turning either right or left. In addition to the delays caused by the difference in speed, the trucks (unless properly provided for) again create serious delays when in the act of loading or unloading. The municipality should encourage all new buildings to provide all-street loading, as well as off-street parking facilities. It should provide places such as loading zones where these vehicles can pull into the curb to discharge or to pick up their loads, without stopping in the second lane. Establishments that are already developed and have not provided off-street loading facilities should be encouraged to confine their loading, if possible, to their alley or their secondary street exits. The municipality should reserve these alleys for such use by the restriction of parking because, not only do they provide the merchants with loading facilities, but they are important fire-protection factors for the buildings in the congested areas. It would not be practical to prohibit the use of streets in the congested area to commercial vehicles, because of the hardship it would inflict on the business establishments. It is possible, however (at least, at certain periods of the day) to restrict certain types of deliveries, such as coal, produce, etc.; and if such arrangement can be made with the express companies, that they endeavor to do most of their delivering prior to noon. From studies made by the Bureau of Traffic Planning Survey, in December, 1934, in Pittsburgh, 48% of the vehicles that stop at the curb are being loaded or unloaded with either passengers or merchandise, which indicates the demand for the aforementioned facilities.

The operation of taxicabs in the congested area creates another form of internal interference during the peak periods of the day. The taxicab, unless otherwise regulated, will travel at a slow rate of speed, because the driver hopes to have a pedestrian hail him for service. This is encouraged by the practice of taxicab companies that compute the wages of their drivers on a percentage basis of their day's receipts. Were the driver proceeding at the same rate of speed as the other vehicles, he is liable to lose this business, as he could not stop suddenly without the possibility of an accident to the vehicle traveling behind him; or he might not see the pedestrian signaling for him. A study of the operation of taxicabs in Pittsburgh showed that 26% were cruising (that is, they approached the same location empty twice or oftener within a 30-min period). The municipality may remedy this interference caused by cruising and slow moving taxicabs by requiring taxicab companies to provide adequate off-street parking facilities, and to see that ordinances be enforced rigidly. However, taxicabs are not the only vehicles that "cruise." Studies made at many intersections in the business district of Pittsburgh showed that in some cases 17% of the traffic on certain streets was cruising traffic (that is, traveling around the block either looking for a place to park, or looking for a person they were to meet, who happened to be late, etc.). To remedy the delay caused by the cruising passenger car is decidedly difficult. This problem has been solved in one or two cases in Pittsburgh by restricting the parking, which eliminates those vehicles looking for a place to park, or by restricting turns at the intersections where cruising is prevalent. These restrictions of turns and parking have made it so inconvenient for the person cruising that many have been discouraged in this practice, and have arranged to meet people in less congested areas.

At wide, uneven intersections having a large street area, where many of the vehicles weave and cross each other, the construction of islands and the "channelization" of the roadway confines the various movements to their proper lanes, and frequently eliminates many of the weaving and crossing delays that formerly existed at the old intersection, in addition to creating a safer condition for both vehicular and pedestrian traffic.

The co-ordination of signals enters into internal interference in that the time for starting and stopping the various types of vehicles differs materially. If the signals are co-ordinated so that the necessity for stopping and re-starting is minimized, many of these delays will be eliminated.

Probably the most serious type of internal interference is due to the loading and unloading of public carriers—the street car and the bus. With the modern improvement of these mass transportation vehicles and the recent development of the model street car, the delay due to different traveling speeds has been greatly reduced. The bus, when stopping to pick up or to discharge passengers, pulls over to the curb and obstructs the curb lane in the same manner as a parked car. The street car does not have the flexibility of being able to pull to the curb to pick up or to discharge its passengers, so that, in many cases, when in the act of loading or unloading, it obstructs not only the curb lane, but also the free lane in which the street car travels. If the adjacent area is controlled by a co-ordinated signal system, signals should be timed so that most of these mass transportation vehicles will arrive at their stops at that period of the signal sequence when traffic in their direction is stopped. That this is practicable is illustrated by the example in Pittsburgh, where it is possible, due to correct signal timing, to do 63.3% of the loading on the stop signals. Another remedy for delays in both lanes by street cars is the construction of street-car loading platforms, which permit vehicles between the loading platform and the curb to move during the time the street car is loading. There has been considerable agitation to pass legislation such that, if a common carrier arrives at an intersection during the green signal, it should not be permitted to stop to pick up or to discharge passengers, but must proceed to the next intersection. This is a drastic remedy and would create a decided hardship for the people who rely on the mass-transportation vehicles.

As previously stated, considerable internal interference has been due to the difference in the speeds of the various types of vehicles and the necessity for the mass transportation vehicles to stop frequently to pick up or to discharge passengers. A possible remedy for this interference is illustrated by the experience in Pittsburgh where two main adjacent thoroughfares, Smithfield Street and Wood Street, were restricted to one-way traffic, northbound and southbound, respectively. The effect was to restrict the parking of automobiles to the right-hand curb lane, and the slow-moving and mass-transportation vehicles to the lane adjacent to the parked vehicles. The success of this plan is illustrated by the lane-ratio data recorded at every intersection along these two main streets, which show that the extreme right lane has a usage of 2.5%, the lane adjacent to the parked vehicles, a usage of 24.5%, the next or third lane from the right, a usage of 60.5%, and the extreme left curb lane has a usage of 12.5 per cent. This indicates that the left half of the street carries 73% of the traffic, whereas the right half carries 27 per cent. If two-way traffic were permitted on these streets, there would be considerable request for permission to park on both sides of the street, and it is probable that street cars would operate in both directions on both streets. These data indicate that making these streets one-way has permitted an increase in capacity of 80% more than if they were operated on a two-way basis. This accomplishment was due to the provision of lanes for the faster moving vehicles; and it does not restrict the capacity of the streets to that of the slower moving vehicles. It is not practicable, however, to confine any street to one-way traffic. Study should be made of the origin and destination of the traffic using the streets, and the possibility of ready access to and from the various destinations. Unless such consideration is given, one-way streets are likely to cause an unnecessary distance of travel and the addition of turns to offset the acknowledged advantages.

Remedies for Delays Due to Medial Interference

The degree of medial interference is largely dependent on the lane widths of the roadways. Where the lanes of a street have a width of not less than 9 or 10 ft, there is little or no medial interference. As the lane width is reduced, vehicles traveling in opposite directions will proceed at slower rates of speed. Medial interference is created when faster moving vehicles desire to overtake and pass other vehicles by pulling around to the wrong side of the roadway. This type of interference can be eliminated readily by the construction of center strip islands, which have yet another advantage in that they can be designed to provide a "reservoir" off the roadway proper permitting vehicles to turn left without obstructing free-moving traffic. The effect of the center strip island is more to eliminate the hazard of accidents than to minimize delays of traffic. It is felt, in general, that such constructions are impracticable in congested districts, because of the additional valuable street area required—an area already limited in many cases. The delays due to medial interference are practically negligible where proper lane widths are available, and it is suggested that concentration be given to the other types of delays because of their seriousness.

OTHER REMEDIES FOR DELAYS

The preceding parts of this paper have dealt with the various types of interference that cause delays to motor traffic. As to the exact seriousness of these delays, little has been published except to evaluate it in terms of units. The relative seriousness is dependent upon the volume of traffic that is involved during certain periods; as this volume increases the congestion will decrease, and as it decreases, congestion will decrease correspondingly. The engineer is faced with the additional problem of analyzing the street system to determine, at no hardship to the inhabitants of the municipality, methods of decreasing the volume of traffic that must travel on the various streets and thus reduce existing congestion.

A possible remedy for reducing the volume of traffic during the peak period is to stagger the business hours. This solution has been discussed for many years and, in some cases, has been found impracticable because of the interlocking of the various types of businesses. It is doubted whether any sincere experiment has been made of the staggering of business hours, because of the habits of the American people and their desire to associate at common hours. For example, when production is necessary, factories and mills operate 24 hr per day; milk companies deliver milk in the early hours of the morning because the public demands fresh milk for breakfast; and meat packers, in order that their products shall not spoil, must deliver their loads before the sun is high. There is no difficulty in the operation of the milk companies or the meat packers, except that there has been a demand that has required the staggering of their business hours. As conditions become more critical and as the various industries realize their loss because of the congestion of traffic, they will undoubtedly favor changing their transportation schedules to more profitable hours. Such a solution of a whole-hearted staggering of business hours is possibly "far-fetched." However, it may be the ultimate solution that the American people must expect, in order to remedy their traffic "snarl." If such a plan were followed, the critical peak period, because of being distributed over several periods, would be materially reduced, and congestion would be correspondingly decreased.

Another possible means of decreasing the volume of traffic is to make a careful analysis and survey of the existing mass transportation system. During the ten years, 1927 to 1936, the private automobile (which carries merely an average of 1.8 persons) has transported about 50% more passengers into a business district than formerly, whereas the total number of passengers carried by mass-transportation vehicles has decreased. This has been due to a change in the mode of transportation of the general public from that of the original mass-transportation vehicle to the private automobile. This change has probably been due to the additional convenience afforded by the private automobile. The engineer of a municipality, whose duty it is to facilitate the movement of traffic safely, should strive to handle the largest number of people in the least number of vehicles. There is little question but that an individual, when driving his automobile to and from work and parking it in the congested area, spends several times the money that he would in using a mass-transportation vehicle. As the mass-transportation facilities are improved, there is little question but that the number of private automobiles in the congested area will be reduced, which will relieve the congestion and also decrease the demand for street and off-street parking spaces.

A survey should be made of the various routes of the surface common carriers: Their speeds, their points of loading and unloading, and the type of their service. There has been considerable controversy over the proper type of routing for surface carriers in a congested area. Three types have been advanced: "Short-looping," "through routing," and general routing. "Short-looping" consists in bringing the common carriers to the outskirts of the business area, discharging and picking up their passengers, and proceeding away from that section. In "through routing," the common carriers proceed straight through the business section, stopping to discharge and pick up passengers as required, and proceeding in the opposite direction from which they originated, to the end of the line, where they turn around and proceed back to town. General routing is the type most commonly practiced to-day. This is the routing of the common carriers through the business section, past the various large department stores, into the center of the section, and turning and proceeding back through the town. This type requires the carriers to make many turns, cross one another frequently, and travel practically a double distance in the congested area. There is no "hard and fast" solution for this condition of the private automobile vying with the surface common carrier. Some of the solutions advanced to the present day involve the construction of elevated lines and subways; and yet the city that does not have these facilities, and must rely entirely on surface common carriers, has a more critical problem. The engineer of a municipality should strive to improve mass transportation facilities, which in the end will decrease the use of the private automobile in the busy areas, and thus reduce congestion.

A third possible solution for reducing traffic volumes is to study and analyze street systems carefully to determine whether adequate by-pass facilities are available to avoid congested areas. From an origin and destination count made in Pittsburgh, indications are that 49.7% of the vehicles entering the business section between the hours of 6:00 A.M. and midnight, left by a different throat from that by which they entered, within a period of 20 min, which time was practically equal to that required to travel from the entry throat to the exit throat. This indicates that the vehicles did not stop in the business section, but, because of not having adequate by-pass facilities, were obliged to use the connection through the down-town section. From further analysis of these entering and leaving data, it is evident that not only does this through traffic hold consistently through the day, but that during the peak period from 4:00 P.M. to 6:00 P.M., 53.3% of the flow in the business section was through traffic, which, if eliminated, would materially relieve the congestion. This is a startling example to illustrate the relief of congestion that would be afforded the City of Pittsburgh if adequate by-passes were provided around the business district.

One should not be misled by this statement, to conclude that facilities are not available in Pittsburgh to enable the avoidance of the congested area, as there are several entry routes that can be used. However, these routes do not have the characteristics that would encourage motorists to use them in preference to the route through the business section. By-pass routes provided should be of a characteristic high-speed design, so as to minimize delays, and should be connected properly to all the entering and leaving arteries. Unless a by-pass has a high-speed characteristic and is attractive to motorists, it will be of little value, and will fail to serve the purpose for which it was created.

Prior to making the "origin and destination" count an engineer making the statement that 53.3% of the flow was through traffic would have been criticized, as it was felt to be an absurd value. Consequently, it is probable that this condition is present in other cities, even if highway officials feel that they have adequate routes for avoiding the business sections; but from a study such as the one cited, the engineer will determine whether these facilities are being used for the purpose intended, and if they are not being so used, an analysis of these by-pass routes should be made to eliminate the delays that are diverting traffic to the congested areas.

Another means of reducing delays falls within the jurisdiction of the police authorities. In preceding parts of this paper, there was discussion of possible restriction of parking, possible restriction of turns, regulation of pedestrian traffic, etc. After a careful study has been made and regulations have been recommended and enacted, it is necessary that they be enforced rigidly, and this is the province of the police. The engineer should be assured of the full co-operation of the police authorities on any regulations established. To enact a regulation does not fulfill the remedy for the delay in traffic; the removal of the vehicle causing the conflict is also required, and it is that conflicting vehicle that must be removed by the police officials.

A municipality should have highly trained police officers, deserving, and having, the respect of the motoring public. Such a staff will facilitate materially the movement of traffic and will be prepared to reduce congestion in the event of an emergency.

Conclusion

To relieve traffic congestion and increase the capacity of a street system is possible, as illustrated in several ways in this paper. Responsibility for the particular type of remedy to be devised is placed upon the traffic and city planning engineers. Their decisions must take many phases into account, such as the habits of the people, the demands of merchants, the cost of the proposed improvements, and the extent of the relief that is to be expected. In many cases, it has been found advantageous, in making such decisions, to temper the recommendations of the engineers by the opinions and suggestions of a group of citizens representing all classes of society, so that by providing an opportunity for every phase of business life to consider the problem, the best possible solution may be obtained. However, the engineers know the causes of congestion and the remedies for increasing the capacity of existing street systems, and any final decisions should be made with their guidance and advice.

IMPROVEMENTS TO REDUCE TRAFFIC ACCIDENTS

By Arnold H. Vey,[11] Esq.

Synopsis

Any detailed analysis of motor-vehicle accident reports discloses numerous conditions, circumstances, and situations involved in the problem of highway accidents. Consequently, hundreds of causes must be considered and, to complicate the problem further, these causes may operate either singly or in combination. Therefore, it is difficult to determine, with any degree of accuracy, the exact number of accidents that may be charged to the car, the highway, or the highway user. All three elements are involved, and they contribute in one way or another, and in varying degrees, to highway accidents; but the highway user (the driver and pedestrian) is responsible for the greatest number. In other words, most motor-vehicle accidents are caused by some improper action on the part of the person or persons involved, regardless of whether such action was wilful, or due to negligence or ignorance.

It seems reasonable, therefore, to suppose that if all highway users knew, and would voluntarily observe, safe highway practices at all times, the present-day accident problem would be materially lessened. Actually, however, the solution is not so simple. Because of human frailties and the impossibility of supervising and governing the actions of all persons at all times, safety on the highway is made extremely difficult. Educational endeavors and increased enforcement activities are not only essential but invariably prove helpful. However, such activities, despite their importance, are not in themselves a complete solution. It is necessary that roadway facilities be designed and applied which are inherently safe, as far as possible, in order to render the failures of human beings of lesser or little importance.

Roadway facilities, of course, are of many component parts, a number of which may be classed as major highway improvements. Time and space will not permit a discussion of all such improvements, and it is the purpose of the writer, therefore, to discuss only those about which accident facts are immediately available. As most major highway improvements during recent years have concerned State highways and, primarily, in rural or less densely populated areas, the scope of this paper is confined largely to improvements on the State Highway System of New Jersey.

Note.—Presented at the meeting of the Highway Division, New York, N. Y., January 21, 1937.

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In this connection, it is important and also of interest to have some knowledge of the accident problem as it exists in New Jersey and as it concerns New Jersey State Highways.

ACCIDENT EXPERIENCE IN 1934, NEW JERSEY STATE HIGHWAY SYSTEM

The data included herein are taken from reports of accidents filed by the motorists involved, together with supplemental reports from police and other officials concerned with motor-vehicle traffic and highway accidents. The reports include accidents involving death and personal injury, as well as those resulting in property damage to the extent of $25 or more.

Total Accidents.—Table 10(*a*) represents the number and severity of accidents in New Jersey during 1934. Of the total accidents, 3.5% involved fatalities, 55.9% involved personal injury, and the remaining 40.6% concerned property damage only; 7 217, or 21.4%, occurred on the State Highway System.

TABLE 10.—NUMBER AND SEVERITY OF ACCIDENTS IN NEW JERSEY DURING 1934

Severity of accidents	(*a*) CLASSIFIED BY TYPE OF ROADWAY						(*b*)* CLASSIFIED BY NUMBER OF LANES		
	State Highways		All Other Roadways in the State		Total		Two lanes	Three lanes	Four lanes
	Number	Percentage of total	Number	Percentage of total	Number	Percentage of total			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Fatal	400	5.5	742	2.8	1 142	3.5	5.05	6.24	6.27
Non-fatal	3 768	52.3	15 069	56.9	18 837	55.9	50.23	54.51	52.59
Causing property damage	3 049	42.2	10 625	40.3	13 674	40.6	44.72	39.25	41.14
Total	7 217	100.0	26 436	100.0	33 653	100.0	100.00	100.00	100.00

* Section (*b*) pertains to State Highways only.

It is interesting to compare this percentage of State highway accidents to total accidents with the percentage of State highway mileage to total miles of roads in New Jersey; for example, of the more than 35 000 miles of improved and unimproved roadways in the State, only slightly more than 1 500 miles, or nearly 6%, represent roadways under the jurisdiction of the State Highway Department. In other words, about 6% of the State roads are the scene of more than 21% of the accidents. Such a comparison, however, is not necessarily a measure of the hazards of State highways, as consideration must be given to the use made of them. Unfortunately, vehicle-mile data, segregated by the jurisdictions of the different classes of official bodies, are not available.

When comparing the severity of accidents on State highways with those on all other roads, it is found that, of the total State highway accidents, those involving fatalities, in percentages, equalled nearly twice the percentage of fatal accidents to total accidents for the remainder of the State, whereas a similar comparison for injury accidents showed a percentage slightly less than that for the remainder of the State. However, if pedestrian accidents are excluded from the results (as most accidents of this class occur in urban or built-up districts), it is found that the percentage of fatal and non-fatal accidents to total accidents is greater on State highways, in each instance, than on the remaining roadways of the State. This is the result of higher average speeds on State highways, of course, and a lesser enforcement of laws and supervision of activities in rural areas than in urban and built-up areas.

A comparison of pedestrian accidents occurring on State roads with those on the remainder of the State, indicates that, of the total pedestrian accidents on State highways, 25% concerned fatalities, whereas for all other roadways, only 6% resulted in fatalities. This, too, is the result not only of greater speeds, but also of the lack of pedestrian sidewalks or pathways along most State highways, as well as lack of proper and sufficient illumination.

Accident Location and Time of Occurrence.—Of the total State highway accidents in 1934, 40.4% occurred at street intersections and the remaining 59.6% at intermediate points. It may be seen, therefore, that the sections of State highway having little or no cross-movements of traffic are the more hazardous. Accidents that occurred at night equalled 46.3% of the total, whereas those occurring during hours of daylight represented the remaining 53.7 per cent. Despite the fact that approximately one-fifth of the 24-hr traffic represents night-time travel, this lesser volume of traffic produces nearly one-half the accidents. In other words, it is more than three times as hazardous to drive at night as during the day.

Accidents per Million Vehicle-Miles.—To afford a comparison of accident experience and hazard on State highways of different widths, an analysis was made of accident rates on those roadways or parts of roadways having two, three, and four improved, hard-surfaced lanes used for the purpose of vehicular travel. Table 11 gives the results of this study.

TABLE 11.—ACCIDENTS PER MILLION VEHICLE-MILES, NEW JERSEY STATE HIGHWAYS, IN 1934

Number of lanes	Million vehicle-miles	Number of accidents	Number of accidents per million vehicle-miles
Two	1 178.78	3 241	2.75
Three	390.85	1 378	3.53
Four	483.21*	1 743	3.61
Total	2 052.84	6 362	3.10

* Includes about thirty miles of divided highway.

From this analysis it is determined that, for the State Highway System studied, the average accident rate equals 3.10 per million vehicle-miles, whereas on two-lane roadways the rate is 2.75, on three-lane roadways, 3.53, and on four-lane roadways, 3.61. In other words, an increased accident rate occurs with an increased width of highways; or (contrary to common belief) the wider the road the more hazardous it is. It is interesting, too, to note that, contrary to common belief, four-lane highways of the undivided type are more hazardous than three-lane roadways.

The writer does not mean by this statement that he supports or endorses the construction of three-lane roadways. However, the facts do prove (at least in so far as New Jersey is concerned) that the rate on four-lane roads is greater by 0.08 accident per million vehicle-miles than on three-lane roadways. There are many possible explanations for this condition, the primary one being that, the wider the road is, the more flexible will be the movement of traffic from one side to the other, unless congestion or enforcement prevents such movements.

A comparison of the severity of accidents on two-lane, three-lane, and four-lane roadways is shown in Table 10(*b*). It is seen that fatal accidents increased slightly with the width of the highway, whereas non-fatal and property-damage accidents fluctuated without regard to highway width. Segregating the accident experience of two-lane, three-lane, and four-lane roadways by accident types, it is found that the percentage of right-angle collisions decreased with the width, as did head-on collisions and fixed-object collisions. Same-direction accidents, however, increased with the width, and opposite-direction accidents (involving a left turn), and pedestrian accidents, fluctuated without regard to highway width. In other words, despite the increase in accident rates with increased width of highways, certain types of accidents conversely decreased, whereas other types increased, and still others fluctuated without regard to width.

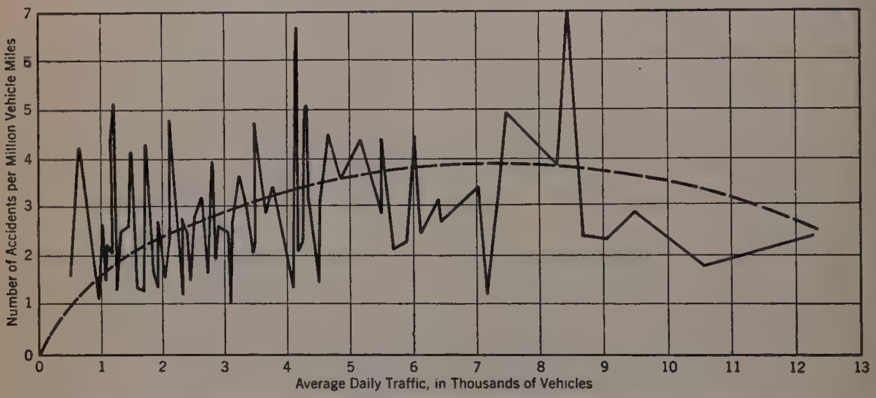


FIG. 4.—RELATIONSHIP BETWEEN DAILY TRAFFIC AND ACCIDENTS PER MILLION VEHICLE MILES FOR TWO-LANE ROADS

Relationship Between Daily Traffic and Accident Rates.—To determine the relationship between daily traffic and accident experience, a survey was made on two-lane roads, or on sections of two-lane roads, in the State Highway System, the results of which are shown in Fig. 4. It is interesting to note the rather definite relationship between daily traffic volume and the number of accidents per million vehicle-miles. In other words, as the average daily traffic increases, accident experience, on a million vehicle-mile basis, likewise increases to approximately 7 000 vehicles per day, which is considered by many authorities to be the capacity of a two-lane highway. Beyond that point, primarily because of congestion and the resultant decrease in speed and flexibility of movement during heavy traffic hours, there is a gradual decrease in the accident rate, despite the increase in traffic.

Types of Accidents.—An analysis of the extent of accidents of various types which have occurred on the New Jersey State Highway System will give an indication as to the probable degree of usefulness of some of the major highway improvements of to-day. Fig. 5 illustrates the relationship of various types of accidents to total accidents, arranged so as to compare those occurring at intersections and between intersections.

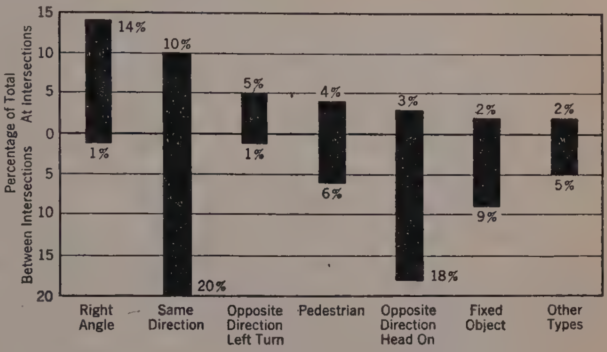


Fig. 5.—Types of Accidents on State Highways

Assuming that it is physically and economically possible to reconstruct the New Jersey State Highway System to the extent of applying major highway improvements now familiar to the highway engineer, including controlled rights of way, in addition to physical separation or segregation of conflicting flows of traffic, adequate shoulders for stopping, pedestrian sidewalks or pathways, proper and sufficient highway lighting, plus other possible advanced features, 75% or more of the accidents that occur to-day could be avoided. In other words, the use of such highway safeguards would make it physically impossible to experience many of the types of accidents indicated in Fig. 5.

For example, a controlled right of way, prohibiting ingress to, and egress from, the highway, except at specific points where there are grade separations, would almost entirely correct the right-angle collision type of accident, as well as collisions between vehicles going in opposite directions with one making a left turn. A center island would prevent most head-on collisions and possibly, too, it might have some effect on same-direction, as well as other, accidents. Pedestrian sidewalks or pathways along major highways would have a materially favorable effect in reducing rural pedestrian accidents.

Furthermore, proper and adequate highway lighting, not only at roadway crossings, but also along sections of heavily traveled highways between crossings, will enable a reduction of all accidents, many of which are of the same-direction type, fixed-object type, as well as the pedestrian type, and which are chargeable to inadequate vision and illumination.

It is impossible, of course, to reconstruct the State Highway System in such a manner without prohibitive expenditures of money. However, much can now be done toward applying some of these so-called highway safeguards or major improvements. Such action will enable a reduction in accidents sufficient to make the necessary expenditures economically proper. Consider, therefore, the accident experience as a result of the application of some of these improvements.

Accident Experience: "Before" and "After" Improvements.—Unfortunately, in most instances, major highway improvements on the State highways of New Jersey were constructed as a part of the initial system and, therefore, reliable "before" and "after" records are not available. In the case of divided roadways, however, it is possible to obtain some rather interesting and reliable "before" and "after" facts for parts of highways that have been converted from an undivided highway to that of a divided type. In the case of other major improvements, and where it has not been possible to obtain "before" and "after" records, interesting comparisons can be made indicating the usefulness of the improvement.

Divisional Islands.—Divisional or center islands that separate opposing flows of traffic, when properly constructed, make it practically impossible for motorists traveling in opposite directions to collide head-on. This type of highway construction has been used in New Jersey for the past several years but, until recently, proper and adequate "before" and "after" studies to determine the usefulness of such construction have not been possible. As a result of converting parts of State Highway Route No. 26 from a four-lane undivided highway to a four-lane divided highway (by sliding one lane of concrete to the side for a sufficient distance to permit the construction of a 12-ft center island, and adding an additional lane to make available a two-lane roadway on either side of the island), it has been possible to obtain "before" and "after" records for a rather limited period. Table 12(*a*) represents a "before" and "after" study on 2.1 miles of this highway in the Township of North Brunswick.

A review of Table 12(*a*) indicates a general reduction of 36% in accidents of all types, after the conversion. It will be noted, too, that not all types of accidents have decreased. Some indicate no change, whereas one type shows an increase. It is important to note, however, that this study—representing not only a rather short section of road but also a limited period of time (not quite five months)—is inadequate to determine with any degree of certainty that the conversion of this roadway to one of the divided type will eventually bring about such a reduction as is now indicated. Nevertheless, it is the writer's opinion that a reduction somewhat comparable to this will be shown as time goes on. Incidentally, all characteristics of the converted highway remained the same as before, with the exception of the construction of the center island as mentioned.

Another example of accident experience after the construction of a center island, but where the roadway was converted from a two-lane highway to a four-lane divided highway, is State Highway Route No. 25 in Cinnaminson and Delran Townships. Table 12(*b*) shows the results of a "before" and "after" analysis of this stretch of highway, representing approximately seven miles of road.

The study indicates a total reduction of more than 46% in all types of accidents, and whereas some types of accidents are reduced, others are increased, and still others remain the same. In this case, however, the reduction

TABLE 12.—Accident Experience, New Jersey State Highways

Type of accident	Experience Before and After Improvement						Collision Record						
	At Intersection		Between Intersections		Total		Daylight				Night		
	Before	After	Before	After	Before	After	Fatal	Not fatal	Property damage	Total	Not fatal	Property damage	Total
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	(*a*) Construction of Center Island, Route No. 26, North Brunswick Township						(*d*) Cloverleaf at Intersection of Routes Nos. 4 and 25, at Woodbridge						
Approaching at right angle	..	..	1	..	1	..	..	..	..	..	..	..	..
Approaching in same direction	..	1	3	3	3	4	..	3	2	5	..	..	..
Approaching in opposite directions:													
Left turn	..	1	1	..	1	1	..	..	..	..	..	..	..
Head-on	..	..	2	..	2	..	..	..	..	..	..	..	..
Fixed object accidents	..	..	2	..	2	..	..	..	..	..	..	..	..
Pedestrian accidents	..	..	2	2	2	2	..	..	..	..	..	..	..
Other types of accidents	..	..	..	..	..	..	..	..	1	1	..	..	..
Total	..	2	11	5	11	7	..	3	3	6	..	..	..
	(*b*) Conversion of Route 25, in Cinnaminson and Delran Townships, from Two-Lane to Four-Lane Divided Parkway						(*e*) Intersection of Route No. 25 and Avenel Street, in Woodbridge Township						
Approaching at right angle	3	3	..	..	3	3	..	1	..	1	..	..	..
Approaching in same direction	1	1	5	4	6	5	..	3	1	4	2	2	4
Approaching in opposite directions:													
Left turn	..	1	..	3	..	4	..	1	1	2	..	..	..
Head-on	1	1	12	..	13	1							
Fixed object accidents	1	..	5	1	6	1	..	2	..	2	..	1	1
Pedestrian accidents	1	..	1	2	2	2	..	..	..	..	..	..	..
Other types of accidents	..	..	..	..	..	..	..	1	..	1	..	..	..
Total	7	6	23	10	30	16	..	8	2	10	2	3	5
	(*c*) Construction of Center Island, Route No. 25, in the City of Elizabeth*						(*f*) Intersection of Georges Road with Routes Nos. 25 and 26, in North Brunswick Township						
Approaching at right angle	7	6	..	..	7	6	..	1	..	1	1	..	1
Approaching in same direction	21	47	4	3	25	50	..	3	3	6	3	3	6
Approaching in opposite directions:													
Left turn	7	7	..	..	7	7	1	4	3	8	1	2	3
Head-on	..	..	2	1	2	1							
Fixed object accidents	3	1	..	1	3	2	..	..	1	1	..	..	..
Pedestrian accidents	10	1	1	..	11	1	..	..	..	..	..	..	..
Other types of accidents	2	6	1	..	3	5	..	..	..	..	..	..	..
Total	50	68	8	5	58	72	1	8	7	16	5	5	10

cannot be entirely credited to the conversion of this highway into a divided roadway as, in addition to the construction of the 10-ft to 12-ft center island, the roadway width was increased from two lanes to four lanes plus a favorable straightening of the alignment in many instances. This study concerns a period considerably longer than that for Route No. 26 as can be noted from Table 12(*b*) and is, therefore, somewhat more indicative of the ultimate results of the improvement.

A further example of the effect of the construction of a center island, but where the improvement differed slightly from that of either of the two previous cases, and where the district is substantially urban rather than rural, is that of Route No. 25 in the City of Elizabeth. Table 12(*c*) shows a comparison of accident experience along 1.3 miles of this highway before and after the improvement. This thoroughfare was initially constructed with narrow islands on each side, approximately 4 ft wide and 20 ft from the curb lines, to segregate local traffic from through traffic. As the roadway width between curb lines was 96 ft, there remained approximately a 48-ft roadway in the center between islands for through traffic.

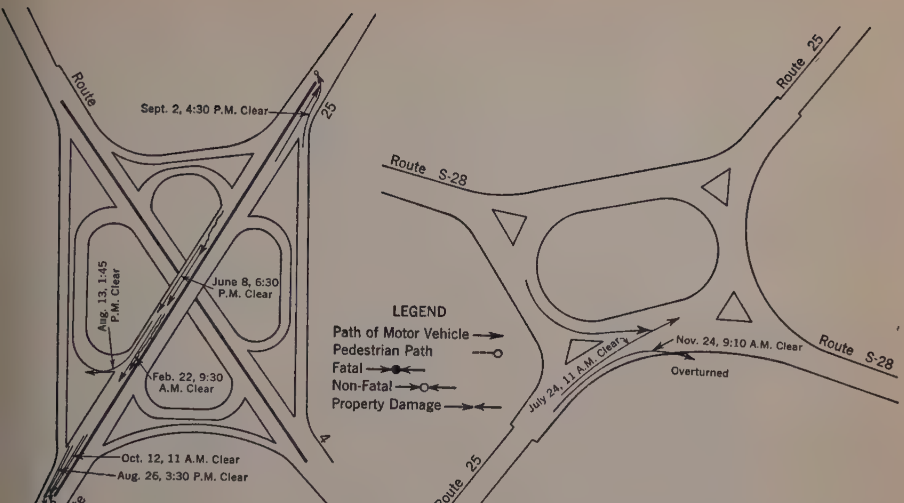
The improvement concerned the removal of the so-called local islands and the construction of a center island, 16 ft wide, down the center of the highway, leaving, therefore, a 40-ft roadway on either side of the center island. A review of Table 12(*c*) shows an increase of about 24% in total accidents after the improvement. Only two types of accidents have increased, however, and one of these types—same-direction accidents, primarily rear-end collisions—increased 100 per cent.

The increase in same-direction accidents is probably the result of improper re-location of signal units at the ten signalized intersections along this stretch of highway after the improvement, in a manner such that motorists traversing the highway can unmistakably receive a signal command sufficiently in advance of the crossing to govern their actions accordingly. It is planned to re-locate signal units properly along this highway, and to improve their operation; and the writer firmly believes that after this further improvement there will be a material decrease in this type of accident. Assuming that a decrease can be accomplished, it will be seen that the improvement of this stretch of highway—eliminating the so-called local islands and constructing a center island—will have a favorable effect on accident experience.

With regard to the improvements at all three of the aforementioned locations, it is interesting to compare the severity of accidents before and after the improvement. For example, on Route No. 26, in North Brunswick, fatal accidents decreased from 2 to none; injury accidents increased from 3 to 5; and property damage accidents decreased from 6 to 2.

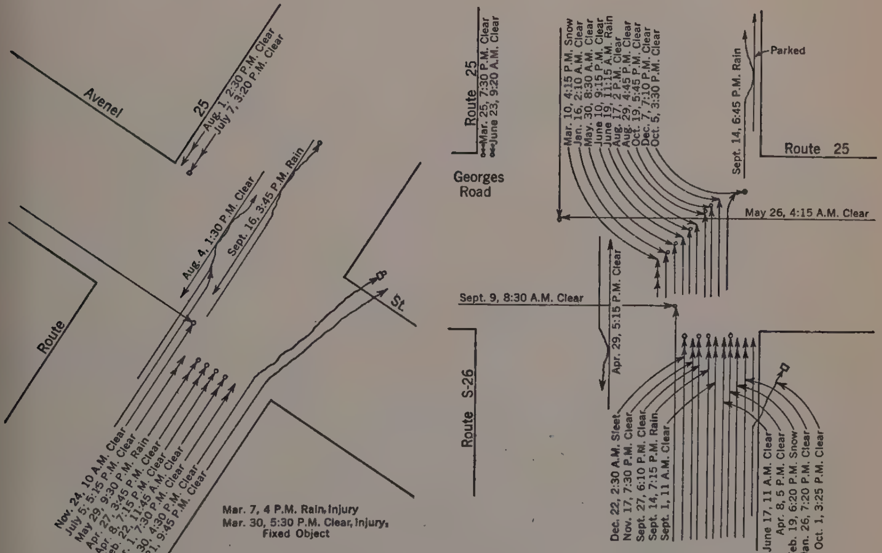
On Route No. 25, in Cinnaminson and Delran Townships, fatal accidents decreased from 2 to none; injury accidents decreased from 14 to 5; and property damage accidents decreased from 14 to 11; and on Route No. 25, in Elizabeth, fatal accidents decreased from 2 to 1, injury accidents decreased from 48 to 19, and property damage accidents increased from 8 to 52.

Grade Separations and Traffic Circles.—Unfortunately, it has been impossible to obtain "before" and "after" records for intersections of highways separated by grade, and at traffic circles. However, it is interesting to make a comparison of accident experience at one of the first grade separations constructed in New Jersey with that of a signalized highway intersection along the same roadway only a short distance to the north; and also at a traffic



(*a*) CLOVERLEAF AT WOODBRIDGE, N. J.

(*c*) INTERSECTION IN NEW BRUNSWICK, N. J.



(*b*) INTERSECTION IN WOODBRIDGE TOWNSHIP, N. J.

(*d*) INTERSECTION IN NEW BRUNSWICK TOWNSHIP, N. J.

Fig. 6.—Collision Diagrams (See Corresponding Data in Tables 10, 11, and 12)

circle along this same roadway with that of a signalized intersection a short distance south of this circle.

Fig. 6(*a*), with Table 12(*d*), indicates the number and type of accidents that occurred on the Woodbridge Cloverleaf, where State Highway Routes Nos. 4 and 25 intersect; and Fig. 6(*b*), with Table 12(*e*), indicates accident experience at the signalized intersection of State Highway Route No. 25 and Avenel Street, a short distance to the north of the Cloverleaf. In this vicinity Route No. 25 averages approximately 25 000 vehicles daily, whereas State Highway Route No. 4 carries about 15 000 vehicles daily, or considerably more traffic than the local roadway intersecting with Route No. 25, shown in Fig. 6(*b*). In other words, despite considerably greater volume of traffic using the Cloverleaf grade separation there were nine less accidents than at the Avenel Street grade crossing for the same period. Of the six accidents at the Woodbridge Cloverleaf three were caused by motorists who, apparently in error, took the wrong roadway or proceeded beyond their turn, and then, in an effort to correct themselves (and in violation of traffic regulations), attempted to make a left turn where such a turn was prohibited.

Figs. 6(*c*) and 6(*d*), with Table 12(*f*), show a similar comparison but between a traffic circle and a signalized intersection. The collision data corresponding to Fig. 6(*c*) are:

Accidents due to approaching at right angles.	1
Other accidents.	1
Total.	2

both cases being property damage only. In this case, the signalized crossing (Fig. 6(*d*)) is on the same highway only a short distance to the south of the traffic circle (Fig. 6(*c*)). The volume of traffic on Route No. 25 in this section equals approximately 13 000 to 15 000 vehicles per day. Despite the constant operation of traffic-control signals at the Georges Road intersection 26 accidents occurred there during 1934. Most of these accidents, however, were the result, either directly or indirectly, of the numerous left turns from the north to the east. Without doubt, a properly designed and constructed traffic circle at this intersection would greatly curtail these accidents.

Highway Lighting.—Unfortunately, reliable "before" and "after" studies enabling a determination of accident experience after the installation of highway lighting, are not available in New Jersey. As a result of a survey made along parts of Routes Nos. 25 and 26 in New Jersey (the first of which is properly lighted with up-to-date equipment, whereas the part along Route No. 26 has no lighting whatever, except at a few isolated points or intersections) some rather interesting facts were developed. These facts were reported in a paper entitled, "Relation of Highway Lighting to Highway Accidents," presented before the New York Section of the Illuminating Engineering Society, in 1934.[12] The conclusions arrived at as a result of this study were, in part, as follows:

> "* * * there is a definite relationship between highway lighting and highway accidents and * * *, in general, where highway lighting is adequately pro-

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[12] Not published.

vided, there is a substantial reduction in night accidents. This reduction in night accidents results in an economic saving in all cases in excess of the cost of providing street lighting, not mentioning, of course, the suffering and sorrow which inevitably accompany motor vehicle accidents."

Other Improvements.—There are many other highway improvements, of course, which may be classed as major improvements and which have a favorable effect on accident experience. Such improvements might include better sight distance at curves and hills, as well as at street intersections; flatter crowns and curves; shoulders wide enough to permit vehicles to park off the normal traveled part of the highway; and the proper use of standard signs, signals, and markings (and these only at locations where they will perform useful functions).

CONCLUSIONS

Safety on the streets and highways depends upon the application of proper "safeguards" as well as the use of proper "safe practices"; that is, unless and until the highways as well as motor vehicles are provided with proper and adequate "safeguards" and the highway user performs "safe practices" when making use of these facilities, the motor-vehicle accident problem will not be solved.

Properly designed and applied, major highway improvements or "highway safeguards" will generally produce a favorable effect upon traffic accidents. In other words, such improvements designed for safety make it less possible for highway users to perform improper practices that cause accidents. Even these improvements, however, are not cure-alls and, with streets and highways as they generally exist to-day, and the impracticability of their immediate re-construction, it is necessary through educational endeavors and enforcement activities to cause drivers and pedestrians alike to practice safe highway habits, as well as to maintain the proper attitude toward the other fellow when on the highways. Such campaigns will do much to curtail the horror and suffering now caused by motor-vehicle accidents. A most fruitful field of educational endeavor to-day is that of teaching the safe use of the highway to young and prospective drivers.

ECONOMICS OF ALIGNMENT, GRADE, AND WIDTH

BY E. L. SCHMIDT,[18] ESQ.

SYNOPSIS

The preceding papers of this Symposium have dealt with causes of traffic congestion and accidents, and have opened a discussion as to necessary remedies. No adequate discussion of this subject would be complete without summarizing the factors relating to cost, and that is the burden of this paper.

INTRODUCTION

The function of a major highway is to connect two or more centers of population and commerce by a roadway and its appurtenances. In the old days, before and during the early stages of the automobile, the guiding principle in considering the relative merits of two or more locations for proposed highway projects was the question of direct costs, and usually the best route was deemed to be that upon which the improvement could be constructed at the least outlay of funds. On the basis of the then existing light volumes of vehicular traffic this probably was not bad economics. As a result of this application of concept, however, most highways at that time followed the "lines of least resistance" to construction problems and rode upon the back of Mother Nature's surface, following contours, hills, and valleys. The consequence, in territory where topography was not kindly subdued, was roads of irregular alignment, heavy grades, and narrow widths.

With the rapid development of the automobile gradually came the realization that costs other than the direct payment of highway construction were involved. The riding public was asked to pay these bills, and rightly so; but if the design of the highway for which this public paid was such as to make him travel longer distances, require greater time, and combat more accident hazards than would be necessary over other possible alternate routes, then this public obviously would be paying proportionately higher costs for automobile operation and probably a greater total price for highway use than would be required with improved and more costly location and development. This condition prompted highway designers to inquire into the question of operation costs. Considering it from this angle alone, the ideal highway would be the one that followed a straight line, was of practically uniform grade throughout, and, of course, provided sufficient traffic-carrying capacity. Such a highway often would not justify the expenditures that would be required to overcome the obstacles of Nature and property developments. Thus, for the conditions of to-day in general, neither extreme is proper. The problem now usually resolves itself into the determination of the most economical highway—the one which will connect the points of termini in the shortest possible traveling distance consistent with reasonable convenience and safety of travel and the economics involved.

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[18] Pittsburgh, Pa.

Highway Economics

The costs involved in highway construction have been designated "direct" and "indirect." Direct costs include all expenses for the actual physical construction of the highway improvement. They consist of the following: (1) Right of way and damages; (2) construction; (3) overhead; and (4) depreciation and maintenance. The indirect costs are: (5) Automobile operation; and (6) automobile accidents.

These costs are not all the factors of value that should be considered in highway improvement problems. Among others, two very important factors to be considered are the values of scenic and beautification advantages, and the values that may accrue from the appreciation of property. The former is an intangible consideration and difficult to evaluate; the latter is more easily measured; but this paper is confined to considerations of highway economics from the point of view of traffic.

Traffic Considerations.—When considering two or more locations and types of development for a proposed highway, the most economical route will be that which requires the least annual expenditure for fixed charges, automobile operation costs, depreciation, and maintenance.

In order to make this determination, it becomes apparent that the starting point in the analysis of all highway problems is traffic. The first question to be answered is "What are the volume and characteristics of traffic likely to be during the life period of the highway improvement?" In the past, this has been difficult to foretell because of the almost unbelievable advance of the automobile. However, it seems that future forecasts will be somewhat easier because primarily the volume of traffic bears a proportional relation to population gain, which is decreasing and probably will continue to decrease to a point of practical stability. Moreover, the ownership of automobiles, now at slightly less than one per five persons, surely must be approaching a "point of saturation." Although the average daily use of cars probably will continue to increase, a practical limit cannot be far remote. Certainly, automobile manufacturers cannot continue forever to install more and more speed in their new models. Their appeal to the wants of car owners, of necessity, will have to be in other directions, such as greater safety and comfort, and in simplifying operation technique so that practically every one can drive with ease. The necessary analysis of traffic demands should be made with these and other considerations in mind.

No general statements can be formulated as to what the average conditions may be because the volume and characteristics of highway traffic are changing continuously from section to section. The only way to make a reasonably accurate determination is by means of a careful and painstaking study of the local circumstances of the problem. In general, however, traffic characteristics may be classified somewhat in accordance with the type of community development through which the highway is traversing. For example, in the commercial districts, practically always, the traffic volume is relatively heavy and speed is restricted. In the urban residential sections, traffic volume is less; but usually it is still high, and the average speeds generally are advanced. In rural residential and further outlying rural sections, traffic volumes have diminished to a relatively small value but, on the other hand, speeds have been materially increased.

Standards of Width, Alignment, and Grades.—It is obviously impossible to establish "hard and fast" general standards in regard to desirable highway designs for these different types of development districts. Every single highway problem must be studied and treated in the light of its own peculiarities, and it can be stated without danger of dispute that no two sets of conditions will ever be found to be exactly the same. Nevertheless, there is usually a general similarity between problems that may arise in similar community sections and an attempt will be made herein to define standards that will fit the rule.

COMMERCIAL DISTRICTS

It is easy to define desirable minimum standards of width, alignment, and grade of major highways that traverse commercial districts, but it is quite another problem to secure the general adoption of such standards. The simplest rule would be to have no major thoroughfares routed directly through commercial sections of any magnitude or importance. Quite often, of course, this rule is not practicable, although there is a decided definite trend toward the by-passing of major thoroughfares around congested business sections. In Pittsburgh, Pa., there is an excellent example of the unnecessary burden of traffic which is superimposed upon city streets because the major thoroughfares leading from outlying districts converge and become a part of the downtown street system. Almost 50% of the local traffic in the business section of Pittsburgh is there, not because of a definite destination in the section, but because it is forced into the "Triangle" due to the absence of adequate by-pass routes. In order to correct this situation, it is proposed to construct a boulevard type of highway along the Monongahela and Allegheny River fronts which, in turn, will connect with a proposed cross-town thoroughfare in the vicinity of the Pennsylvania Railroad Station in Pittsburgh. The cost of this river-front improvement probably will be approximately $5 000 000. No doubt it will be justified time after time, however, by the value of accommodation which it will afford traffic.

In a commercial district, where there may be no choice but to utilize the existing street system for major highway use, attention should be directed toward selecting those streets that are best adapted for the purpose. Wherever possible, streets should be chosen that will afford convenient movement for through traffic with the least interference to local traffic. Of course, no rule can be made for this determination. It can only be solved by a study of the conditions at hand. However, in a commercial district, the hourly traffic almost always will exceed the working capacity of a two-lane roadway for moving vehicles and, therefore, wherever possible, such streets should be selected as will provide capacity for four moving lanes—two in each direction—as well as adequate provision for parking spaces. Generally, six lanes for moving vehicles on one street within the limits of a single roadway, is not advisable, even if there may be evidence of congestion on the four-lane highway. Where it appears that there is need for additional traffic capacity, it is probably better to devise methods by which part of the traffic may be diverted into other streets.

In commercial sections, because of interference caused by traffic on intersecting streets, parked vehicles, pedestrian movement, street cars, and the other usual factors, automobile velocity generally is quite low, probably varying from 10 to 25 miles per hr and averaging about 15 miles per hr. At this range of speeds, the question of alignment does not present any troublesome problems. As long as there are no awkward effects, movement of traffic would not be retarded nor accident hazards increased because of short radius curves. At street intersections, it is desirable, if possible, to provide minimum radii of approximately 50 ft in order that large buses and trucks may turn without causing interference in other lanes of traffic.

The maximum grade in commercial districts, where justified economically, should not exceed 4 per cent. In old established and concentrated business sections, highway improvements, such as providing better alignment, reduced grades, or increased widths of roadways, usually will require enormous direct costs due to the high values of property along the right of way. Ordinarily, in order to justify these expenditures, the advantages that will accrue from the improvements must be of considerable magnitude.

Urban Residential and Other Outlying Districts

Outside the commercial districts, traffic characteristics will vary considerably, and desirable minimum standards of alignment, grade, and width of highways will depend upon the conditions peculiar to the problem in question. In general, it may be stated that the desirable curvature, gradient, and width of roadway, depend upon the volume and speed of the traffic that probably will use the highway, as well as the general topography of the territory in which the improvement is to be made.

Physical Characteristics

Widths of Roadways.—The determination of roadway widths is fundamentally a study of roadway lane capacity. Ordinarily, the definition of lane capacity has been interpreted as the maximum number of vehicles that can pass a given point on the highway, in a given time, at specified speeds. This measure of capacity, an arithmetical computation, has been determined frequently and it has been found possible, without interferences, to pass 2 000 cars per lane per hr, at a speed of 20 miles per hr. This finding, however, has no real practical value since there is no consideration given to the speeds that drivers would naturally be inclined to follow in the absence of limitations imposed by traffic, and no clue as to the point where congestion may be said to begin. The latter is an extremely important consideration since it determines, more or less, the time when attention should be given to the question of highway capacity improvement.

A better definition of capacity was given by the Highway Research Board, National Research Council, at its Eighth Annual Meeting in 1931, as follows:

"The traffic capacity of a roadway is reached when any further increase in traffic volume, all other factors remaining constant, results in a marked decrease in traffic speed." This interpretation of traffic capacity is significant because it indicates the point at which any additional traffic will begin to result in retardation of traffic movement. Bruce D. Greenshields, Assoc. M. Am. Soc. C. E., appropriately designates such capacity as the "working capacity" of the highway. As the result of experiments conducted by him in 1934, on Ohio State highways, by which traffic volumes and speeds are recorded and correlated, he reaches the conclusion that the average automobile driver (if not most drivers) will operate his car at a speed of about 44 miles per hr, whenever traffic conditions permit him to drive as he sees fit. Professor Greenshields designates this velocity as "free speed" and finds that it obtains a density of approximately 400 cars per hr on a two-lane highway, with a varying percentage of truck traffic to 10%, and with traffic about equally divided in both directions. Whenever traffic exceeds this density, velocity begins to decrease. Apparently, however, the decrease in velocity could not be called marked until a density of approximately 600 to 700 cars per hr is reached.

These experiments "dovetail" with the arithmetical computations made by Sigvald Johannesson, M. Am. Soc. C. E., who computes "working capacity" of a two-lane roadway as approximately 200 vehicles per hr per lane. At this density, vehicles are likely to be spaced so as to permit passing in both directions with reasonable safety.

The writer substantiates these findings by his local experience and observations. It is his conclusion that the working capacity of a two-lane roadway approximates 600 vehicles per hr, and he would recommend that whenever it is found or estimated that the average daily density of traffic exceeds this value for 3 hr or more of the day, consideration should be given to the economics of widening.

In the Pittsburgh area, a maximum hourly density of 600 cars on a two-lane highway (as determined by a comprehensive traffic survey conducted in 1934 under the direction of John M. Rice, M. Am. Soc. C. E.) is equivalent to a total volume of approximately 5 000 cars per day. It is interesting to note the comparison between this volume and the standard of practice for road design established by the American Association of State Highway Officials. This group recommends a roadway capacity of more than two traffic lanes in cases where the average daily volume exceeds 4 000 vehicles.

Two lanes for moving vehicles, of course, is the minimum for any highway. Whenever traffic conditions indicate the need of widening a road, or the construction of a new highway of greater capacity than two moving lanes the change should be from a two-lane to a four-lane roadway. Only in exceptional cases should the widening from two to three lanes be considered. Although the addition of one lane will increase capacity very materially, it usually will convert the highway into a most dangerous thoroughfare.

A highway with four lanes for moving traffic will carry a large volume, probably (under favorable conditions) as great as 50 000 cars per day without appreciable congestion. Where more than four traffic lanes seem to be needed to take care of the traffic, most likely it will be found better to construct an additional thoroughfare.

Because of the seriousness of head-on collisions, the present tendency is to advocate the construction of a divided roadway with two or three traffic lanes for each direction instead of a four-lane pavement. This type of development, certainly, is desirable and, wherever economically justified, should be provided. The width of separation between roadways, where provided, will depend upon the character of local conditions and may vary from a few feet to as many as several hundred feet or more. Wherever possible the minimum distance between directional roadways, at least where cross-overs are provided, should be 40 ft. This will permit vehicles to turn from the inside lane of one roadway to the inside lane of the other without obstructing the traffic movement in other lanes of travel. Where there is considerable truck or slow-moving traffic, consideration should be given to the desirability of providing, physically or by regulation, separate lanes of travel for this traffic.

Curvature.—The most important reasons for curvature and variations in alignment are: (*a*) To make the highway serve intermediate points between the principal termini; (*b*) to decrease damage, right-of-way, and construction costs by the utilization of existing locations, or by taking advantage of favorable topography or other natural conditions; and (*c*) to reduce the rate of grades by lengthening the distance of travel. Except as it affects distances, curvature does not appreciably affect the cost of motor-vehicle operation, provided proper radii and length, super-elevation, sight distances, and roadway widening are made part of such curvature. The desirable magnitudes of these factors depend mostly upon the rate of speeds at which traffic will move normally. On a two-lane roadway, the standards should be more liberal than those for a four-lane highway where directional traffic is confined to its own lanes.

The desirable minimum standards of curvature and sight distances are as follows:

Item:	Desirable minimum standard:
Sight distance, in feet	800
Horizontal curvature; radius, in feet	1 000
Vertical curvature; sight distance, in feet	500

In each case these are desirable minimums and should be exceeded wherever practically possible. However, in territory such as Western Pennsylvania, because of the rugged terrain and topography, it sometimes becomes expensive, and not economically justified.

Current practice in Allegheny County is to superelevate all curves on a two-lane roadway which are in excess of 1° curvature. As a rule, the superelevation is fixed at $\frac{1}{8}$ in. per ft of width per degree of curvature, with a maximum superelevation of 1 in. per ft. On four-lane highways, superelevation usually is provided, but the extent and design ordinarily are determined by special study of each individual case rather than by the results of fixed formulas.

Grades.—The rugged topography in the Pittsburgh territory does not lend itself to the construction of grades as low as those in many other sections of the United States. Engineers in the region, of course, attempt to keep grades as low as practicable and try never to exceed a maximum of 8 per cent.

Costs and Savings

Direct Costs.—The sums that are expended for the direct cost items in the improvement of highways vary materially, and it is difficult to determine the exact effect which alignment, grade, and width have on these items. Costs of rights of way and damages are the most variable, and in the writer's experience have ranged from $15 000 to $100 000 per mile. In commercial districts, obviously, this item is usually high and may constitute the major part of total direct costs; or, in fact, in many instances, it may cause the cost of a proposed improvement to be prohibitive, making it necessary to construct a highway along an alternate route. In sections outside commercial districts, this item of cost decreases with the intensity of development and the extent of property values.

In Allegheny County, a fair average cost for 60-ft rights of way and damages in districts outside of commercial and highly developed residential areas, is about $25 000 per mile. For a 100-ft right of way, the corresponding cost probably would average about $40 000 per mile, provided no material building developments were encountered. These averages give a reasonable measure of the probable difference in cost of rights of way for a highway in Allegheny County with two lanes and four lanes for moving vehicles.

Cost of construction does not cover the wide range of variation per mile of highway as rights of way and damages; and, unless influenced by peculiar or unusual construction problems, such cost is practically the same in the different types of community districts. For the type of development customary in connection with two moving vehicular traffic lanes, construction costs in Allegheny County average about $75 000 per mile. Where four lanes for moving vehicles are provided, construction costs average about $140 000 per mile. Thus, on the average, the cost of constructing a four-lane highway is about 1.86 times that for a highway with two lanes for moving traffic. This is a measure of the effect of width on the direct costs of major thoroughfares. This additional expense, of course, is offset by reduced automobile operation costs.

As already stated, the effect of introducing curvature and grades into a highway usually is to reduce construction costs. However, where it becomes desirable, or necessary, or economically justifiable, to reduce grades or improve alignment on existing highways, the effect is to provide a highway of greater direct cost than the existing one that is being replaced.

Cost of overhead (being composed of expenses for contingencies, engineering, organization, interest, office, legal, and similar items which bear a relation to both rights-of-way and construction costs) is affected, therefore, by the range of variation in these two cost items. Overhead costs usually are expressed as percentages of the total estimated cost of the improvement. For conditions in Allegheny County, overhead costs average about 15% of the total estimated costs of rights of way and construction, and on the basis of the average values given for these items, would equal about $15 000 to $27 000 per mile for highways with two and four traffic lanes, respectively.

Cost of maintenance is a continuously recurring charge in contrast to the other direct cost items, which are non-recurring. Maintenance varies principally with the construction cost item and reflects expenditures for keeping the highway in proper condition for traffic use. Obviously, it varies mostly with width of pavement, but alignment and grades also affect this cost item since there is more "wear and tear" and need for conditioning pavements as the sharpness of curvature and grades increases. Experience with high-type road construction in Allegheny County indicates that the annual cost of maintenance, covering all items for a two-lane roadway, averages about $1 000 per mile, and about $1 200 per mile for four-lane highways.

Indirect Costs.—Various attempts have been made to evaluate the benefits to traffic that accrue by the reduction of travel distances, traffic delays, and traffic danger hazards. Except for the question of traffic hazards, the bases of such estimates are automobile operation costs, and include the items of gasoline, lubrication, tires and tubes, maintenance, depreciation, license, garage, interest on investment in car, insurance, taxes, and sometimes driver's wages. In a study made by Thomas R. Agg and Harold S. Carter, Members, Am. Soc. C. E., in 1928,[14] it was found that the cost of operation of the average passenger automobile which traveled 11 000 miles per yr, was 5.44 cts per mile when operating on high-type road pavements. Mr. Johannessen concludes[15] that the average operation cost of passenger cars used for private purposes at that time amounted to 7.5 cts per mile, based on an annual 10 000-mile use; that the cost for passenger cars used for business purposes on the same annual mileage basis was 12 cts per mile; and that about two-thirds of the passenger cars are used for pleasure and one-third for business. In a similar manner, other estimates have been made for the cost of truck operation and the results have been found to vary from approximately 20 to 70 cts per mile, depending upon the size and tonnage capacity.

In Allegheny County, as the result of various traffic surveys, the average conditions seem to be as follows:

Passenger traffic (percentage)	80
Pleasure purposes (percentage)	70
Business purposes (percentage)	30
Truck traffic (percentage)	20
Average truck capacity, in tons	1.5
Average annual mileage	12 000

Based upon these averages the writer's estimates of automobile operation costs are as shown in Table 13. These unit costs are the bases for the following evaluation of benefits arising from highway improvements which affect reductions in travel distance, traffic delays, and danger hazards.

Saving by Reduction in Distance.—Distance affects both operating costs and time, but consideration is given herein only to the economics of the former, as the value of time will be considered separately under the heading, "Reduction of Traffic Delays." The values in Columns (2) to (5), Table 13, will be affected directly by any reduction of distance, assuming that ample roadway capacity is provided for traffic, but the total annual sums spent for the other values will not be changed to any significant extent.

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[14] *Bulletin 91*, Eng. Experiment Station, Iowa State Coll., Ames, Iowa.

[15] "Highway Economics," by Sigvald Johannessen, M. Am. Soc. C. E., 1931.

These operation costs become 3.75 cts for passenger automobiles (pleasure or business purposes), and 9.83 cts for trucks. Based on a distribution of 80% passenger cars and 20% trucks, the average cost per vehicle amounts to approximately 5 cts per mile or about 1 ct per 1 000 ft. At this rate the annual savings due to a reduced traveling distance of 1 mile for 5 000 cars per average day amounts to $91 250. This is equivalent to a capitalized cost (at 6% interest) of $1 520 000, and is a measure of the sum of money that can be expended justifiably to reduce the traveling distance for an average daily traffic of 5 000 vehicles by 1 mile. Expressed in other terms, these results indicate that under the conditions assumed, the annual expenditure of about $57 500 will be justified to reduce the travel distance 1 000 ft per 1 000 vehicles per day. Among other methods, such reduction of distances can be accomplished by improving the alignment and reducing the grades.

TABLE 13.—ESTIMATE OF AUTOMOBILE OPERATION COSTS, IN CENTS PER MILE (12 000 MILES ANNUALLY)

Vehicle	Gasoline	Lubrication	Tires and tubes	Maintenance	Depreciation	License	Garage	Interest	Driver's wages	INSURANCE		Total
										Fire and theft	All accidents	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Passenger:												
Pleasure	1.33	0.36	0.40	0.83	1.66	0.13	0.42	0.50	....	0.10	0.83	6.56
Business	1.33	0.36	0.40	0.83	1.66	0.13	0.42	0.50	5.00	0.10	0.83	11.56
Trucks	3.00	0.63	1.10	3.00	4.20	0.40	0.50	1.50	7.50	0.25	1.25	23.33

Savings by Reduction of Delays.—The value of savings in traffic delays has been estimated as from 1 ct to 2 cts per vehicle-minute. Under the conditions that prevail in Allegheny County a fair and conservative value is 1.25 cts. This may be supported upon an analytical basis either by placing a reasonable value upon the rider's time or by expressing the fixed charges on a minute-of-time basis. However, an actual practical basis has greater meaning and, therefore, the writer refers to such an example in the Pittsburgh District. For a part of the suburban South Hills Section two routes are available for travel to downtown Pittsburgh, one of which, between common points, is 5.7 miles long and the other is only 4.0 miles. However, there is considerable traffic congestion on the shorter route and the average travel time for the 4 miles is about 13.5 min, whereas the longer route can be traversed in 9.5 min. Because of the greater convenience of travel and the shorter time required for those to whom both routes are available, drivers of passenger cars invariably select the longer one. This means that in so doing, preference is given to spending about 6.5 cts more for operation costs in order to save 4.0 min. On this basis the value of a vehicle-minute would be 1.6 cts. Considering all the circumstances involved in this example, and the theoretical computations used, 1.25 cts is a conservative value. Its significance is that wherever a highway improvement—either by providing greater width, better alignment, reduced grades, by-passing of congested areas, or by any other means—results in a reduction of travel time, the value of this reduced time is equivalent to 1.25 cts per vehicle-minute under the average conditions that prevail in Allegheny County. Using the example of 5 000 vehicles per average day, the annual savings of 1 min per vehicle would equal \$22 800, or an equivalent 6% capitalized cost of \$380 000. On the basis of 1 000 vehicles per day this would be \$76 000.

Savings in Reduction of Accident Hazard.—The economic loss caused by traffic accidents in the United States has been estimated at about \$3 000 000 000 per yr. On the basis of 10 000 miles per vehicle, this is equivalent to an average accident-hazard value of about 1 ct per vehicle-mile. This is a general measure of a value that may be applied where reduction of accident hazards is accomplished by highway improvement. Of course, a better method is to study the accident proneness and the costs of the particular highway or section of highway under consideration and, thereby, derive a value that will be more applicable to the conditions at hand.

In a general presentation of this kind, it is practically impossible to evaluate savings due to reduction of traffic accidents and, therefore, no consideration is given to such values in the following specific examples.

Examples

The effects of alignment, grade, and width on the direct and indirect costs of major highways cannot be equated, varying as they do for each individual highway improvement. In general, however, curvature is introduced into a highway to reduce rights-of-way, damage, or construction costs. The introduction of such curvature does not materially affect vehicular operation costs in so far as the reduction of traffic speeds are concerned, provided such curvature is of proper radii, length, and superelevation, and if it provides good sight distance. If it increases traveling distances, curvature obviously increases vehicular operation expenses.

The economics of grades is difficult to evaluate. Where grades are introduced in order to fit topography, the effect is to reduce construction cost. Provided such grades do not exceed a rate of about 6%, and if there is sufficient width of roadway and sight distances, the cost of vehicular operation does not become a particularly significant factor. On the other hand, where heavy cuts and fills are made in order to reduce grades to desirable standards, or to decrease the distance of travel or the need for curvature, the effect is to increase construction costs materially but, compensatingly, to decrease vehicular operation costs. The provision of additional width causes a slight increase of rights-of-way and damage costs and a heavy increase in construction costs. On the other hand, however, such improvement increases vehicular speeds and, consequently, lessens vehicular operation costs.

Case 1.—The application of the economics of highway improvement can best be illustrated by reference to specific examples. In 1936, a study was made of a proposed major highway extension in the Pittsburgh area. The proposed improvement was about 3 miles long and it was estimated that the average annual traffic during the next 20 yr would approximate 2 000 000 cars per yr. The problem resolved itself into two alternatives, one of which would adapt the location of the highway to topography, involving curvature and grades, and would make the line about 3.6 miles long. The other location was more direct (about 3.0 miles long) but would involve considerable cut and fill and the construction would cost about twice as much as the longer route. The economics of this problem were as shown in Table 14. Project B, the shorter but more costly project in so far as direct costs apply, is the more economical.

TABLE 14.—ECONOMICAL COMPARISON, ROUTES A AND B

Project	Length, in miles	Total physical cost, in thousands of dollars	ANNUAL COSTS, IN THOUSANDS OF DOLLARS					
			Interest at 4%	Depreciation	Maintenance	Vehicular operation		Total
						For distance, at 5 cts per mile	For time, at 40 miles per hour	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	3.6	500.0	20.0	12.5	3.6	360.0	135.0	531.0
B	3.0	1 000.0	40.0	25.0	3.0	300.0	112.5	480.5

Another excellent example is the Liberty Tubes. These are twin vehicular roadways, about 1⅛ miles long, constructed by tunneling through a steep hill known as Mount Washington, so as to connect down-town Pittsburgh with the South Hills (residential) District. Although they are exceptional in character, the tubes are nothing more than a four-lane highway. However, the construction cost of this highway was more than $5 000 000 per mile. Nevertheless, in overcoming obstacles of topography to eliminate excessive curvature, grades, and distances, this expenditure has been justified over and over again by the value of the accommodation to traffic. It has resulted in a daily savings of both time and distance to about 25 000 cars, which is equivalent to more than 9 000 000 vehicles per yr. Without recourse to any computations, it is apparent that this convenience is easily worth 10 cts per vehicle. At this rate, the annual savings to traffic is $900 000. Capitalized at 6%, the worth of this savings is $15 000 000. On the other hand, if this improvement had only affected, in the same manner, 1 000 vehicles per day, the annual savings would be only $36 000 which, capitalized, would represent approximately $600 000. Under the latter condition, it would be obviously impossible to justify the $5 500 000 construction cost. Incidentally, this particular improvement is an outstanding example of one which has resulted in a tremendous appreciation of property values. In one township alone, in the South Hills area, affected by the construction of this project, the assessed value of property has increased from $8 000 000 to $29 000 000 within 10 yr after its opening in 1924.

Case 2.—One other splendid example of the economics involved by improvement of alignment, grade, and width, is the so-called Westinghouse Cut-Off. This is a relocation improvement of the Lincoln Highway in an area immediately outside the city limits of Pittsburgh. Between common points, the old route, 3.5 miles long, provided capacity for two moving lanes. Of the total 3.5 miles approximately one-third followed an indirect route through a heavily congested commercial district and the remaining two-thirds was routed over a hillside, resulting in an average rate of grade of about 10%, and several extremely sharp curves. The average time for traffic to travel this old course was 14 min.

The new highway, following an entirely new location, is 2.5 miles long, has a capacity for four lanes of moving vehicles and provides a practically uniform grade of 4 per cent. To accomplish these improvements it was necessary to make very heavy cuts and fills, the maximum of each exceeding 120 ft. The total physical costs amounted to almost $3 000 000, of which 42.5% was for rights of way and damages. The average travel time over the new route is 4.3 min.

The annual charges for interest at 6%, depreciation, and maintenance, amount to $333 000. Offsetting these charges, the annual costs of automobile operation have been reduced at least $425 000 due to the value of the savings in time and distance to the 2 500 000 vehicles at present utilizing the highway.

Conclusion

It is impossible to present general formulas of the effect of alignment, grade, and width on the direct and indirect costs of major highways. This applies particularly to direct costs because of the extreme variations of physical conditions throughout the United States. This paper has outlined the procedure followed and the experience gained in Allegheny County. Because of the ruggedness of the terrain, the problems are much more difficult, and the direct costs heavier, than in most other sections. This results in greater expenditures for value received. In practically all cases, however, the engineers concerned have been able to justify the expenses for major highway improvements.

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DISCUSSIONS

ANALYSIS OF STRESSES IN SUBAQUEOUS TUNNEL TUBES

Discussion

By Orrin L. Brodie, M. Am. Soc. C. E.

Orrin L. Brodie,[7] M. Am. Soc. C. E. (by letter).[7a]—Simplification of analyses for obtaining the most probable stresses to be expected in subaqueous tunnel tubes is always welcome and the author has presented his method of analysis clearly and briefly. His paper should prove serviceable to all engaged in this kind of work.

The application of the Maxwell theorem of reciprocal displacements, while not novel, is extremely apt in connection with the feature of the restraining effect of horizontal tie-rods. Mr. Eremin is correct in his conclusion that the use of this theorem, in this respect, affords a simplification over that of the method of least work; but the writer has applied the latter to tunnel stress analysis, by means of a simple graphic adaptation of the method, which served as a satisfactory check to the more precise method by deflections. The use of the elastic center for the origin is also simpler, in the deflection method, than placing the origin at the invert, as has been done in studies of this kind. Tabular methods of computation always should be used; but after the basis of computation has been established by the theory, it is doubtful whether or not, in general, the subsequent labor of computing varies much. As the author has stated, the mechanical part is a matter of preparing tables and diagrams.

Although it is obvious that the theoretical derivations, such as those outlined in the paper, are independent of the physical conditions surrounding the structure, nevertheless the paper arouses curiosity along the lines of the assumed loadings for the tunnel considered. Some developments of tunnel stress theory have incorporated the "external loading" in the integrations. The author very properly has omitted these, as they only encumber the analysis unnecessarily, especially as the use of unit

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Note.—The paper by A. A. Eremin, Assoc. M. Am. Soc. C. E., was published in December, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1937, by David J. Peery, Jun. Am. Soc. C. E.

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[7a] Received by the Secretary October 14, 1937.

forces and moments suffices in this respect; but as the main objective of any analysis is to make practical application toward evaluating the stresses in the structure for proper design, it must be admitted that some appraisal of the external forces forms the other half of the "picture."

No doubt it is true that each tunnel project presents conditions in this respect peculiar to itself; but the writer is out of sympathy with the thought that because the determination of external forces is often more or less uncertain, in an absolute sense, theoretical studies, such as the author has presented, are of no practical value. In each undertaking more is learned, and uncertainties of yesterday become cleared up to-day. The value of theoretical investigation is that by its application in a comparative manner, the experience gained by the construction of works successfully accomplished may be carried over to present problems of projected work. The theory provides the vehicle. In other words, theoretical analyses permit of comparing tunnels of different sizes and types under approximately like conditions. In order to emphasize this the writer ventures to review, as briefly as possible, the main results of the Holland Tunnel studies, after which he will illustrate one phase of this work by considering the example shown by the author in Fig. 5.

The comparative studies were made mainly on the Pennsylvania Tunnel tube, as being most similar in kind of material, along with the tunnel tube considered, with others suggested. Extensive stress analyses were made by means of the method of least work and the method of deflections. Studies of the local stresses at the joints, both end and side, were based upon the results of these analyses. The cast-iron ring containing the entire internal structure and the cast-iron ring alone (as it would be during construction), with and without compressed air, were subject to the investigation. The depths over the top of the tunnel ring which gave the probable, greatest loadings, were 31 ft of silt beneath 38 ft of water, and the weights of silt and water were taken, respectively, at 104 and 64 lb per cu ft.

Due to the wide variations in the behavior of the silt of the Hudson River, as had been observed by different people at different times and under varying conditions, diverse opinions existed as to the actual forces that might be expected to be active externally. This is explainable if one considers the noticeable variation in the consistency, composition, water content, and weight of silt, after the violent disturbance of the shoving of the shield and then after a considerable time during which it has become quiescent around the tube. To include material of varying consistency, Rankine's theory of earth pressure was applied, through varying "angles of repose," although it was well recognized that that theory did not include the tenacity, or cohesion inherent to a more or less determinable degree, in the river silt. (Subsequent experiments disclosed a value reaching 0.4 lb per sq in., with a water content of 55% by volume. This value was indicated by a computation to contribute toward offsetting the tendency to rise because, by the Rankine theory, the result was that in some cases the weights of the tunnel ring alone, with the overlying material bearing upon it, were less than the upward, active pressures of the silt.)

Where "floatation" was indicated, due compensation was made by the distribution of a uniform load either at the top or bottom, as required, before proceeding with an analysis.

During construction, silt pressures were observed around the tube over a considerable period of time, by specially designed pressure gages, the tubes of which were let through the grout holes of the ring castings. Concurrently with these observations weight and water content determinations were made. The silt weight ranged from 93 to 114 lb per cu ft and the water content, from 52% to 64% by volume. Laboratory tests followed on transmission of conjugate pressures, whence as an index, angles of repose were computed. These angles varied between 1° 45′ and 2° 40′. The pressure gages in the tunnel disclosed that these angles were less at the top than at the bottom of the ring. The recent advance in the realm of "soil physics" gives promise, along these lines, of clarifying much that, in the initial stages of the Holland Tunnel construction, was more or less speculative and rendered the minds of some engineers skeptical of theoretical analyses.

In mentioning the Holland Tunnel, the author referred to high tensile steel in the tie-rods. It is believed that this tunnel was the first to have the tie-rods introduced. The tunnel ring bolts were also of the same material. It might be of interest to note in this connection that the idea for the use of high tensile steel in the Holland Tunnel, for this purpose, was proposed originally in December, 1920, by Ralph Smillie, M. Am. Soc. C. E., then Designing Engineer on the Tunnel Staff. There were several other features of design besides the bolts and tie-rods which were developed by the comparative studies made possible by the analyses. The segments of cast iron and steel were thickened at the corners of their flanges, which insured better castings and reduced local stresses in the flanges. It had been demonstrated that changing the location of the bolts connecting the end flanges of the ring segments, by moving them nearer to the inner sides of the web and side flanges of the segment, resulted in a more rigid joint and, consequently, a stiffer tunnel ring. Furthermore, it was proved that the lower middle bolt, then customary in end-flange design, was entirely inutile; but only with the introduction of the high tensile steel and corresponding decrease in bolt diameter were these innovations made possible. The bolts were tightened to an initial stress of 25 000 lb per sq in. and both the saving in bolt and tie-rod metal, together with a great reduction in the necessity of bolt tightening after each shove of the shield, amply justified the modifications in design. The effect of the tie-rods was approximated by supplemental calculations, which indicated a reduction in flexure in the ring by about one-half to nearly two-thirds, from that without their restraining effect.

The interest in the "loading" applied by the author was aroused by Fig. 5, in his exemplification of the application of the equations, illustrating the writer's contention of the affinity of the conditions surrounding a tunnel to stress analyses. Fig. 5, however, indicates that the example loading is fluid in nature, or that the Rankine angle of repose is zero. Furthermore, it is presumable that the dead weight of the tube is equal to the difference between the upward and downward vertical forces. For this reason the writer would be interested to learn of the actual conditions of loading assumed by the author, if those given do not rightly reflect them

TABLE 1.—Comparison of Maximum Moments in Tunnel Rings

Tunnel	For a Load, W_1, in Pounds per Cubic Foot, of:	
	53.7	104
Angle of repose, ϕ	0°	3°
Posey	24 500	20 000
Cast iron	1 100	36 000

Table 1 serves to compare the moments in the Posey Tube between Oakland and Alameda, Calif., with one of cast iron, of the same outside diameter (37 ft), first under the loading inferred from Fig. 5 and then under the weight of material surrounding the Holland Tube as assumed for that tunnel, and with an angle of repose of 3 degrees. As a measure for comparison, the maximum bending moments for each have been computed, without tie-rod restraint, by the following equation:

$$M = w\,D\,a + \left(1 - k - \frac{D}{H_s}\,b\right)\frac{W_1\,H\,D^2}{c} \qquad (31)$$

in which: M = bending moment (in foot-pounds) in a tunnel ring 1 ft long; w = weight of tunnel, in pounds per linear foot; D = external diameter of tunnel ring, in feet; H_s = depth, in feet, of the center of the tunnel ring below the surface of the silt (the depth of water above is reduced to an equivalent depth of silt, which is added to the actual silt); W_1 = weight of 1 cu ft of silt or material, in pounds; a, b, and c are constants; and:

$$k = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad (32)$$

in which ϕ = the angle of repose of the material surrounding the ring. Values of the constants, a, b, and c, for use in Equation (31) are given in Table 2. It should be borne in mind that Equation (31) yields only the value (not the location) of the maximum bending moment. A negative moment in an answer signifies only a reversal from the moments that have a plus sign.

Fig. 5 indicates that the specific gravity of the material appears to be only 0.86. The head on the center of the 37-ft tube is 89.5 ft; its weight is 73 500 lb per lin ft, the weight of material around it, 53.7 lb per cu ft, and, the angle of repose, first 0° and second, 3 degrees.

The weight of a cast-iron tube, correspondingly, should be about 56 000 lb per lin ft. The results in Table 1 indicate that the Posey Tube sustains less flexure in the heavier cohesive material, whereas the cast-iron tube fares better in the liquid. No doubt it would float; but this has not been considered in the example.

TABLE 2.—VALUE OF CONSTANTS FOR USE IN EQUATION (31)

Angle of repose, ϕ	a	b	c	$1 - k$	Angle of repose, ϕ	a	b	c	$1 - k$
0°-0′	+0.0361	0.1084	4.00	0.000	7°-30′	0.0343	0.4706	17.20	0.230
2°-0′	−0.0236	0.0974	5.41	0.067	8°-0′	0.0363	0.4875	17.42	0.244
3°-0′	−0.0247	0.1295	6.94	0.101	8°-30′	0.0377	0.4995	17.50	0.258
3°-30′	−0.0120	0.1758	8.65	0.117	9°-0′	0.0387	0.5080	17.50	0.270
4°-0′	−0.0078	0.2620	11.26	0.130	9°-30′	0.0394	0.5145	17.41	0.283
4°-30′	0.0102	0.2824	12.00	0.144	10°-0′	0.0372	0.5030	17.15	0.295
5°-0′	0.0232	0.3740	15.03	0.160	10°-30′	0.0349	0.4920	16.75	0.309
5°-30′	0.0275	0.4125	16.20	0.176	11°-0′	0.0326	0.4805	15.97	0.321
5°-45′	0.0263	0.4040	15.96	0.182	12°-0′	0.0327	0.4862	15.83	0.344
6°-0′	0.0411	0.5305	18.90	0.189	13°-0′	0.0328	0.4919	15.70	0.368
6°-30′	0.0359	0.4835	17.85	0.204	14°-0′	0.0329	0.4975	15.57	0.390
7°-0′	0.0307	0.4440	16.86	0.218	15°-0′	0.0330	0.5030	15.43	0.411

Equation (31) was derived and constants computed as an epitome, as it were, of the studies alluded to by the writer. It may be used directly for fairly close approximations of values for tunnels of different sizes and weights and depths of submergence. Curves in terms of angles of repose and the moments as arguments may then be plotted and comparisons made.

The maximum bending moment usually occurs at the crown or invert although it has been found at the haunches, depending upon conditions, and occasionally between the crown and the haunch. Moments at the haunches are of opposite sign to those at the crown and invert, and the usual assumption is to consider moments to be positive when producing compression at the "extrados" of the ring, and negative if they produce tension.

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DISCUSSIONS

DEFLECTIONS BY GEOMETRY

Discussion

BY DAVID B. HALL, ASSOC. M. AM. SOC. C. E.

DAVID B. HALL,[11] ASSOC. M. AM. SOC. C. E. (by letter).[11a]—In a method of solving problems for which other solutions already exist, it is natural to inquire into the relative advantages of the proposed method. The writer is naturally pleased to observe that those who have discussed the paper have expressed emphatic, favorable views on this question, finding that the proposed method makes the subject of deflections more clearly and easily understood. Mr. McGaw and Professor Grinter, who are both teachers, have especially stressed its value for beginners on this account.

There appears to be some skepticism, however, as to the possibility of this geometrical method taking its place beside some of the older methods for ordinary routine computations, and also for solving difficult problems. It is the writer's feeling that such advantages as the older methods may have are not inherent, but must represent developments with which the newer method has not "caught up." After all, any substitute method depends for its existence upon some mathematical relationship, and however varied its physical interpretations may be, its mathematical operations must be substantially the same. One tool that Professor Grinter mentions, the Williot diagram, in reality, should be classed as one of the most legitimate of the methods of the geometry of deflections.

To make the geometrical method comparable with the older methods as a practical working tool, the greatest need is evidently an accumulation of working details. Several valuable contributions are to be found in the discussions.

Mr. McGaw has begun this filling in by translating the law of reciprocal deflections into geometric terms; and in doing so he has probably made

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NOTE.—The paper by David B. Hall, Assoc. M. Am. Soc. C. E., was published in December, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1937, by Messrs. A. J. McCaw, and L. E. Grinter; April, 1937, by Ralph W. Stewart, M. Am. Soc. C. E.; and May, 1937, by William Bertwell, Assoc. M. Am. Soc. C. E.

[11] Haworth, N. J.

[11a] Received by the Secretary October 25, 1937.

this law more intelligible to many readers. Perhaps it should be noted that one additional step is necessary to develop, fully, a formal proof along this line, because the law of reciprocal deflections applies to any structure in which the materials follow Hooke's law, whereas the foregoing proof is based on relations that exist only in a statically determinate structure. The additional step is merely to trace the effect of adding a redundant support or member to a simple structure.

Specifically, consider three points, *A*, *B*, and *C*, on a simple beam. A redundant support is to be added at Point *B*. When a load is applied at Point *A*, the beam will deflect at Point *B*. If a support is added, the reaction will be proportional to this deflection. Finally, this reaction will produce a deflection at Point *C*. Since, in the simple beam, the reciprocal relation is known to exist between Points *A* and *B*, *B* and *C*, and *A* and *C*, all the factors, multiplied together to find the deflection at Point *C*, due to a load at Point *A*, appear in finding the deflection at Point *A* due to a load at Point *C*. Finally, if the principle applies for a structure with n redundants, the foregoing proof shows it to apply for $(n+1)$ redundants, thus completing the proof by mathematical induction.

A useful geometrical conception, essential to any comprehensive treatment of the subject, is Mr. Stewart's method of traversing the elastic curve. Although it may take a little time to grasp this method fully, the speed and simplicity with which it may be applied to a wide variety of problems make it well worth mastering.

In presenting two alternative solutions for the unsymmetrical beam of Fig. 10(*c*), Mr. Stewart and Mr. Bertwell have illustrated, effectively, the readiness with which solutions may be improvised by these methods—in contrast, for instance, to the necessity for following, faithfully and blindly, some of the older standardized forms for arch computations.

In the second half of his discussion Mr. Bertwell has made a number of observations pertaining to an intriguing branch of mathematics—approximate integration. In its elementary form this is the problem of constructing a polygon to take the place of a curve (as in Fig. 7, for example), and determining the location of the polygon with respect to the curve. In Fig. 7, this polygon consists of a series of tangents to the curve. In such a case Mr. Bertwell's proposal to use interpolated values, which are at the points of tangency of the polygon and curve, is sound. An alternative refinement, used at one time by the writer, was to make corrections at the vertices of the polygon in the same manner that the middle offset of a vertical curve is computed. This correction is $\frac{1}{8}\,\alpha\,dx$. Thus, in Table 1, the correction at Point 10 would be $\frac{1}{8} \times 2.96 \times 4.4 = 1.6$; and the correct deflection would be, $420.6 - 1.6 = 419.0$.

The arrangement in Fig. 10(*c*) and in Tables 2 and 3, however, presents the following entirely different approach, which Mr. Bertwell failed to recognize. If a curve is drawn and marked off at equal intervals, and a polygon is then drawn consisting of a series of chords joining these points, then the angle at any vertex of the polygon will be the product of the curvature in its vicinity and the length of one interval; and the angle between the last chord and the end tangent will be one-half the product of the curvature there and the length of an interval. In this manner the desired results were obtained directly with no extra operations.

It is believed that further refinements, when they are warranted, may be developed more easily in the direction of integration by Simpson's rule, than along the lines described by Mr. Bertwell.

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DISCUSSIONS

STRUCTURAL ANALYSIS BASED UPON PRINCIPLES PERTAINING TO UNLOADED MODELS

Discussion

By David M. MacAlpine, Assoc. M. Am. Soc. C. E.

David M. MacAlpine,[29] Assoc. M. Am. Soc. C. E. (by letter).[29a]—In so far as the author has introduced the little used idea of *f*, this paper is an interesting one. It is unfortunate, however, that he has chosen an unfamiliar sign convention. For the paper to realize its greatest value to the profession it is imperative that this convention be clearly stated in the closing discussion. It will then be possible to transform the formulas developed to more familiar ones.

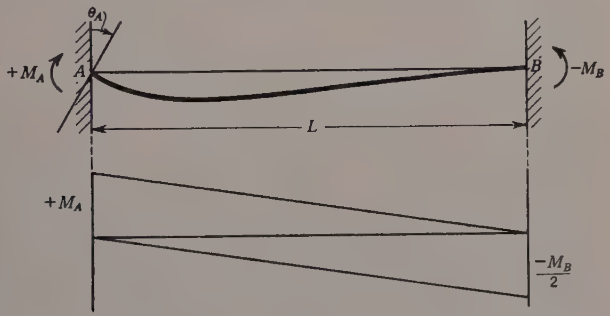

Fig. 24.

There appears to be no need of abandoning any of the several rather widely accepted and publicized sign conventions. One wonders why all these conventions were discarded. If the author had accepted the rather common convention of tension on the bottom fiber being caused by positive moment, Equation (36) would not have been changed. A derivation of this formula, without recourse to the slope-deflection equation, follows:

Note.—The paper by Otto Gottschalk, Esq., was published in January, 1937, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: March, 1937, by Messrs. L. J. Mensch, and Frederick Shapiro; May, 1937, by Messrs. James R. Griffith, and Camillo Weiss; and September, 1937, by Messrs. F. B. Farquharson, and A. A. Eremin.

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[29a] Received by the Secretary September 7, 1937.

Holding End B fixed (see Fig. 24) and rotating End A so as to produce tension in the bottom fiber:

$$M = - M_A + \frac{3}{2} M_A \frac{x}{L} \qquad \text{(69a)}$$

and,

$$E I \frac{dy_1}{dx} = - M_A\, x + \frac{3}{4} M_A \frac{x^2}{L} + C_1 \qquad \text{(69b)}$$

When $x = L$: $\frac{dy_1}{dx} = 0$; and, $C_1 = \frac{M_A\, L}{4}$:

$$E I\, y_1 = - \frac{M_A\, x^2}{2} + \frac{M_A\, x^3}{4\, L} + \frac{M_A\, L\, x}{4} + C_2 \qquad \text{(69c)}$$

When $y_1 = 0$: $x = 0$; and, $C_2 = 0$:

$$y_1 = \frac{M_A\, x\, (x - L)^2}{4\, E\, I\, L} = \frac{M_A\, x\, (x')^2}{4\, E\, I\, L} \qquad \text{(69d)}$$

When $x = 0$: $E\, I \frac{dy_1}{dx} = E\, I\, \theta_A = \frac{M_A\, L}{4}$; $\theta_A = \frac{M_A\, L}{4\, E\, I}$; and $f' = \theta_A\, L$;

$\frac{M_A}{4\, E\, I} = \frac{f'}{L^2}$; and,

$$y_1 = \frac{f'\, x\, (x')^2}{L^3} \qquad \text{(70a)}$$

Holding End A fixed and rotating End B so as to obtain tension in the bottom fiber:

$$y_2 = \frac{f\, x^2\, x'}{L^3} \qquad \text{(70b)}$$

Adding Equations (70*a*) and (70*b*), the result is identical with Equation (36) of the paper.

On the other hand, the beam convention is very inconvenient to apply to the slope-deflection equation. A sign convention must fit some definite system if it is to be used from a practical, as well as from a mathematical, standpoint. Such a convention would require that positive angles be measured by counter-clockwise rotation, that distances, forces, and displacements be positive upward and to the right from the origin, and that moments capable of being represented by a positive vector be positive; that is, counter-clockwise external moments in the plane of the paper are positive. In this case positive moments are increasing positive angles. No great harm is done, however, if any other consistent convention is used.

In this convention, the familiar slope-deflection equations are:

$$M_{AB} = M_{F\,AB} + \frac{2\, E\, I}{L} (2\, \theta_A + \theta_B - 3\, R_{AB}) \qquad \text{(71a)}$$

and,

$$M_{BA} = M_{F\,BA} + \frac{2\, E\, I}{L} (2\, \theta_B + \theta_A - 3\, R_{BA}) \qquad \text{(71b)}$$

Ordinarily, $M_{F\,BA}$ is arithmetically negative but should carry a positive sign in the formula. Using Equations (71) and deriving formulas similar to Equations (34*b*) and (34*c*):

$$m = \frac{-\,0.5\,k}{k + S_B} \qquad \text{(72a)}$$

and,

$$m' = \frac{-\,0.5\,k}{k + S_A} \qquad \text{(72b)}$$

These expressions are in accordance with the geometric facts, θ_A being opposite in direction to θ_B. Equations (37*a*) and (37*b*) become, respectively,

$$\theta_{BA} = -\,\frac{S_2}{K_B} \qquad \text{(73a)}$$

and,

$$\theta_{BC} = \frac{S_1'}{K_B} \qquad \text{(73b)}$$

Equations (73) are also in accord with the geometry of the figure. The remainder of the equations follow directly, differing from the author's formulas only in sign.

Referring further to signs, the author has failed to state that he assumes L to be negative when measured to the left of his origin. This is necessary in order to obtain Equation (16) in its present form. This assumption, however, introduces an interesting incongruity in Fig. 2(*a*). For example, both θ_A and θ_B are positive by the author's sign convention. The values of f and f' are then positive and negative, respectively, thereby violating the sign convention as adopted for the paper for L. It is necessary to assume L as positive when measured to the right for θ_A and to the left for θ_B; otherwise, f' is positive when measured upward and f is positive when measured downward.

Referring to Line 2 following Equation (2), the use of the word, "strain," rather than "moment," is open to question. It would seem that "moment" is the more exact term.[29b]

The derivation given in Appendix I does not seem to be as satisfactory as the many other derivations of Maxwell's theorem to be found in any text or reference book on the subject. The theorem is so well known and widely publicized that there is some question as to the necessity of a derivation unless a new and better proof is given.

The derivation of the slope-deflection equations, using any of the accepted sign conventions, is also familiar and Appendix II only serves to illustrate the breakdown of the author's sign system (see Fig. 14).

There is a definite need for the establishment of a mathematically consistent, and practical, sign convention, as well as notation, in both the beam theory and stress analysis. Any sign convention that is not self-consistent tends to confusion and delays the establishment and acceptance of a correct one.

[29b] Corrections for *Transactions:* In the line directly following Fig. 4, change "single" to "simple"; and, in Equation (41*b*), in the left-hand member, change $\frac{\text{"}m'\text{"}}{\Delta}$ to $\frac{\text{"}f'\text{"}}{\Delta}$.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

THE PASSAGE OF TURBID WATER THROUGH LAKE MEAD

Discussion

BY MESSRS. J. C. STEVENS, AND C. S. JARVIS

J. C. STEVENS,[42] M. AM. SOC. C. E. (by letter).[42a]—In calling to the attention of the profession one more interesting phenomenon with which the silt problems of the West are replete, the authors of this paper have "started something."

The silt problems in the West are manifold. How and where does silt originate? How is it transported? What space does it occupy in reservoirs? What part do the cultural activities of Mankind play? What steps can be taken to hold it intact in the basins? Can it be prevented from entering reservoirs, or can it be removed therefrom once it has entered?

The phenomenon so interestingly described by the authors is only one of a great number of silt problems that should be "run to earth" and made controllable in some degree by human agencies.

The records of 1935–36 indicate that during moderate and low-water periods of inflow, a slug of fine silt brought into the reservoir by a tributary may remain intact and pass entirely through the reservoir. At Lake Mead these slugs appeared to originate in the basin of the Little Colorado River. There is no reason to doubt that the same part could be played by the San Juan River or by the Virgin River if a summer cloudburst could produce sufficient run-off with an attendant quantity of fine silt.

The spring floods on both the Colorado River and Rio Grande originate from melting snows in the upper reaches of the water-shed. The sediments at such times are predominately coarser than those on the lower tributaries. In June, 1935, the average daily load passing Grand Canyon was about 1 800 000 tons per day of which only about 400 000 tons, or

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NOTE.—The paper by Nathan C. Grover, M. Am. Soc. C. E., and Charles L. Howard, Esq., was published in April, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1937, by Messrs. O. A. Faris, Paul A. Jones, Carl E. Scofield, and Ivan E. Houk; September, 1937, by Messrs. William P. Creager, Harold K. Palmer, Morrough P. O'Brien, John C. Page, John H. Bliss, and B. H. Monish; and October, 1937, by Messrs. D. M. Forester, A. D. Lewis, G. C. Dobson, and William W. Rubey.

[42] Cons. Engr. (Stevens & Koon), Portland, Ore.

[42a] Received by the Secretary September 20, 1937.

22%, was less than 20 microns in diameter. The flood of September 29 to October 5, however, brought in a maximum of about 2 000 000 tons per day, of which 1 800 000, or 90%, was less than 20 microns in diameter. The latter produced noticeable silt discharge whereas the former did not.

It is hoped that more data will be published regarding the silt flow through Elephant Butte Reservoir. The papers by Messrs. Lawson[5] and Fiock,[6] referred to by the authors, do not give details. The writer is of the opinion that if the silt flows of the past were compared with up-stream summer floods it would be found that most of these silt flows follow floods on Rio Puerco and Rio Salado, and, as in the case of the Colorado River, they will not coincide with the spring floods which carry the coarser material.

There appears to be no doubt that the phenomenon of silt discharge results from an intimate mixture of water and fines of such coherency that the specific gravity of the entire mobile prism is increased until it flows along the lowest channel of the reservoir, quite independent of the temperature differences between the reservoir and flowing water. The slowness with which fluids of different densities mix has been well demonstrated by some experiments on both liquids and gases made at the University of California. Diffusion among mixtures of varying densities was found to be surprisingly slow.

It appears that the stream of water-silt only issued from the reservoir through the tunnels with entrances at stream level. Since the tunnels were closed, no silt has been discharged, indicating that, at the dam, this mixture has been unable to rise the 250 ft necessary to flow through the penstocks or the outlet works.

Although the total quantity discharged was only 2.5% of that coming into Lake Mead in 1935, the fact that any at all was discharged is too important not to be investigated further. These fines probably would not occupy nearly so much space in the reservoir as an equal weight of coarser material; nevertheless, the question arises as to whether it might not have been worth while to have left gates in the tunnels that could be opened at such times, instead of having them plugged permanently. On other reservoirs, the proportion of silt that could thus be discharged might have greater significance and would raise the question as to the wisdom of maintaining stream-level outlets.

Another phase of this problem concerns the quality of the water passing through a reservoir in this manner. Such a concentration of fines may render the water unfit for irrigation on certain soils as it makes them "slick" and effectively seals them against air and water drainage. Prolonged use of such concentrates or intermittent use over long periods of time is quite certain to be detrimental to almost any type of soil; hence, from this standpoint, it might be better not to permit such silt slugs to pass through a reservoir.

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[5] "Movement of Silt, Elephant Butte Reservoir," by L. M. Lawson, *Reclamation Record*, Vol. 10 (1919), p. 411.

[6] *Transactions*, Am. Geophysical Union, 1934, Pt. II, p. 472.

Each basin must be studied on its own merits in these regards, in which the geological, chemical, mechanical, and physical factors must be analyzed carefully. All these problems of silt are of tremendous importance to the West and it is "high time" that an orderly, well-planned approach to their solution was begun without further delay.

In its report on "Drainage Basin Problems and Program, 1936," the National Resources Committee included as one of its principal study projects a thorough investigation of the silt problem in the Colorado River Basin, and recommended an early allotment of $1 000 000 with the understanding that several million dollars would ultimately be required for an extended period of research.

In order to secure co-ordinated results, it would seem essential that such an investigation be headed by an agency or board composed of the chiefs of the several Federal bureaus now engaged, independently, in investigating some phase of the silt problem, as well as a number of non-Federal engineers and scientists, to the end that the elements of engineering, hydrology, chemistry, plant biology, geology, and soil mechanics will be adequately represented and that co-ordinated conclusions will be secured.

The answers to some angles of this problem are being sought independently by certain Federal agencies, among them the Soil Conservation Service, the Forest Service, the U. S. Geological Survey, the Bureau of Reclamation, the Bureau of Agricultural Engineering, the Bureau of Plant Industry, and the Grazing Division. There is little or no co-ordination of effort among these agencies, however. A co-ordinating board would concern itself solely with the research phase of the problem and would enlist the services of all agencies concerned, including the several hydraulic laboratories. To the latter could be assigned special problems, while the field research could be conducted by Federal agencies. Such an investigation of the silt problems in the Colorado River Basin would be equally valuable for the Rio Grande Basin, and all other silt-producing basins in the Arid West.

C. S. Jarvis,[43] M. Am. Soc. C. E. (by letter).[43a]—Experience with operation and maintenance of numerous small reservoirs had led a number of engineers, including the writer, as early as 1914, to expect the passage of heavily silt-laden water as a separate stream through the clear water of large reservoirs. Regarding the small reservoirs as hydraulic models, they concluded that large reservoirs should respond to the treatment so commonly used for maintaining the capacity of small storage projects in arid regions, where both the erosion rate and the water values are relatively high. As early in his career as 1897, the late Arthur P. Davis, Past-President, Am. Soc. C. E., had proposed the use of such expedients as hydraulic giants and sluicing for the removal of such silt.[44]

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[43] Hydr. Engr., Soil Conservation Service, Washington, D. C.

[43a] Received by the Secretary September 30, 1937.

[44] "Irrigation near Phœnix, Arizona," by Arthur P. Davis, *Water Supply Paper No. 2*, 1897, pp. 81–83.

Serious consideration was being given in 1914 to the plans for installing a flushing conduit to extend several miles up stream from the dam to the narrowest section of the storage basin, with a view to trapping the sediment and conveying it through the deepest part of the reservoir to the outlet. The prevailing opinion among the consultants was that the heavier, silt-laden inflow would follow the valley trough and would convey a considerable portion of the suspended load to the river channel and irrigation canals, thereby prolonging the life of the storage project and adding fertility to the irrigated land.

A corollary to the axioms recognizing the impoverishment of soils through erosion would acknowledge the values in soil fertility to be restored through desilting reservoirs by sluicing and spreading the suspended load upon the land surface.

In the writer's discussion[45] of "The Silt Problem of the Zuni Reservoir," a plan was outlined for maintaining a turbid outflow to keep pace with, and correct, the silting process. At that time (1920), the passage of turbid water through Elephant Butte Reservoir, if observed at all, had not been widely heralded. It is recognized that disadvantages would attend the silt-laden outflow, due particularly to increased wear of turbines and to silting of canals supplied from the reservoir.

Computations in connection with Fig. 2 disclose the fact that the occasions when turbid water was discharged from Lake Mead were a few days after heavily silt-laden inflow had occurred at Grand Canyon, with suspended matter representing 2% to 5% by weight. By inference, one may assume that it requires a pronounced difference in specific gravity of inflowing and impounded water to build up the inertia or momentum sufficiently to maintain the unmixed flow through the reservoir. As described by Mr. Fiock,[6]

> "Observations have revealed that the silt-laden water flows along the bottom of the lake in a very thin sheet (less than 5-ft depth), or through the lowest portions of the floor of the Reservoir much as water itself flows under air, and apparently does this without diffusing into the water already in the lake, as it retains certain distinct characteristics which are evident and have been observed as it is discharged from the outlets."

If the 6 000 000 tons of silt discharged from Lake Mead during 1935 represent 2.5% of the average annual load brought into the lake, then the total annual silt inflow is about 240 000 000 tons, or nearly 660 000 tons per day, or 7.6 tons per sec. On the basis of 16 000 000 acre-ft of annual inflow, or nearly 22 600 cu ft per sec, the weight of average inflow, in tons per day, is 61 517 000. Therefore, it appears that the average load of suspended matter represents $\frac{660\,000}{61\,517\,000} = 1.07\%$ of the weight of water.

If it is necessary to maintain 2% or more of silt content in order to insure turbid flow through the lake, it would require some kind of turbulence, such as may be produced by dredging pumps or hydraulic giants, to displace the required 7 tons or more of sediment per sec, during the intermittent periods of operation. Obviously, for such conditions, the dredging would continue not more than one-half the time to keep pace with sedimentation, or proportionately less for partial benefits.

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[45] *Transactions*, Am. Soc. C. E., Vol. 83 (1919–20), p. 883.

[6] *Transactions*, Am. Geophysical Union, 1934, Pt. II, p. 472.

Let it be assumed that suitable types of floating dredging plants have been provided to agitate and to bring into suspension one-fourth of the annual silt inflow, or 60 000 000 tons, representing nearly 2 tons per sec throughout the year, but capable of greater output during actual working hours. Also, assume 1 000 000 acres as susceptible of irrigation from Lake Mead (about 500 000 acres in Imperial Valley, 100 000 acres in the Yuma Project, and 200 000 acres in Mexico, a total of 800 000 acres in 1936), the major portion of 60 tons per acre per yr would be en route to enrich the irrigated land.

If the amortized cost of installation, together with maintenance and operating expense for the necessary floating plant and mechanical equipment, should amount to $2 000 per day, or $730 000 per yr, the charge per acre served would be only $0.73. The actual value of the fertile elements delivered with the turbid water should be several-fold more than the cost, while other benefits realized would be the prolonged usefulness of reservoir capacity, and the checking of accelerated channel erosion below Boulder Dam resulting from the recent clarification of effluent.

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DISCUSSIONS

PRACTICAL USE OF HORIZONTAL GEODETIC CONTROL

Discussion

BY R. M. WILSON, M. AM. SOC. C. E.

R. M. WILSON,[10] M. AM. SOC. C. E. (by letter).[10a]—Many questions that arise in the use of horizontal-control surveys are included in this interesting paper. One of the most important of these is: In what form should the results of control surveys be expressed to meet the requirements of the many kinds and classes of dependent surveys? The basic net of first- and second-order triangulation, reaching into all parts of the United States, provides a rigid framework by which to determine the correct horizontal position on the earth's surface of all other surveys that are connected with it.

It is well understood that the geographic co-ordinates of a triangulation station identify, exactly, the position of that station on the surface of the earth. It is true that the rectangular co-ordinates of that station also identify its position exactly, provided the rectangular co-ordinate system is based on a formal plane projection of the geodetic reference system.

The standard rectangular co-ordinate systems for the various States developed by the U. S. Coast and Geodetic Survey are based on mathematically accurate principles. They provide the means to transform geographic co-ordinates into rectangular co-ordinates, and *vice versa,* for any particular point in a survey, without in the least jeopardizing the identity of the position to which the co-ordinates refer. It is apparent, therefore, that co-ordinates in either form represent equally well the position of a point as determined by geodetic surveys. One form is merely a translation of the other. It should be noted that the results of first- and second-order triangulation contained in the more recent *Special Publications* of the

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NOTE.—The paper by R. C. Sheldon, Assoc. M. Am. Soc. C. E., was published in April, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1937, by Messrs. Philip Kissam, Ralph Z. Kirkpatrick, E. B. Roberts, H. W. Hemple, J. C. Carpenter, and George D. Whitmore.

[10] Chf., Section of Computing, U. S. Geological Survey, Washington, D. C.

[10a] Received by the Secretary October 16, 1937.

Coast and Geodetic Survey give the co-ordinates in terms of latitude and longitude and also in terms of x and y referred to the standard State systems.

It is left to the discretion of the user, therefore, to decide which form of co-ordinates will best suit his particular requirements. It is not at all necessary, however, that an engineer must accept one form for all surveys to the total exclusion of the other. In general, geographic co-ordinates are desirable in projects that are geographic in scope, that is, those that involve large parts of the earth's surface. Rectangular co-ordinates are the natural form in which to express detailed and intensive surveys within small areas, and surveys made in connection with the development of local engineering projects.

Mr. Sheldon mentions the U. S. Geological Survey as one of the organizations using geodetic co-ordinates. The topographic "quadrangle" maps issued by the Survey are integral parts of the topographic map of the United States, and it is logical that the map should be subdivided according to uniformly spaced meridians of longitude and parallels of latitude. The units are thus arranged in an orderly and systematic manner throughout the country from Maine to California and from Florida to Minnesota. There is no discontinuity in arrangement because of State lines. It is obvious, therefore, that geographic co-ordinates are the best form in which to express the positions of control-survey stations used in the standard mapping operations of the Geological Survey, because this mapping is geographic in scope. The system just described was selected by the Geological Survey before 1880. Even if the State rectangular co-ordinate systems had been available for use at that time, any system based on them could not have been considered as comparable in usefulness to the one actually chosen and in use to-day for subdividing the topographic map of the United States.

The Geological Survey recognizes the value of the State rectangular co-ordinate systems, however, and is now publishing many of its standard topographic maps with marginal marks to show where the grid lines of these systems may be drawn. Any engineer can then draw the grid lines, and so plot upon the map, or scale from it, any positions expressed in rectangular co-ordinates in which he may be interested.

The kind of transit traverse described by Mr. Sheldon, with its results computed in terms of geographic co-ordinates, is not new. The Geological Survey began using it for the control of topographic mapping as early as 1893. During the 20 yr since 1916 a total of about 100 000 lin miles has been done, and in 1935 alone the Geological Survey executed 9 204 lin miles of transit traverse. The field and computing methods have become more accurate and efficient with the passing of time, so that these control surveys have remained consistent with the increasing accuracy of the mapping based on them. Frequent azimuth observations are made by the simple astronomic methods familiar to surveyors to avoid any appreciable accumulation of azimuth error in the traverse.

The conversion factors given by Mr. Sheldon in Table 2 are the reciprocals of the *M* and *P* factors used by the Geological Survey to express the results of its traverse in the form of geographic co-ordinates. The proved usefulness of these surveys justifies the attention that Mr. Sheldon has given them.

The standard co-ordinate systems developed by the Coast and Geodetic Survey are designed to be State-wide in application. Nevertheless, any individual engineering project or survey that involves an entire State is extensive enough to be considered geographic in scope, so that geographic co-ordinates ordinarily would be appropriate to use for it.

In surveys extending over an entire county, or over a compact group of several counties, rectangular co-ordinates are convenient to use. Then, because the systems are continuous throughout large zones in each State, any county survey usually will be consistent and continuous with surveys made in adjacent counties. Discontinuity exists only along State boundary lines and along the lines separating the large zones; but the systems provide an overlap, so that no ordinary survey need be separated into parts because it lies across a State or zone line.

Many county surveys, and nearly all city surveys, are already referred to some rectangular co-ordinate system. Most surveys for general engineering projects, for mining or other local projects in which accurate and intensive work is required, use rectangular co-ordinates. The State systems provide the means to express the results of these surveys in common terms, so that they may blend harmoniously and so that each may have the benefit of the results of others adjacent to it. To use these systems no real change is needed in the methods already employed in plane surveying beyond accepting the co-ordinates for the origin and the azimuth of an initial line as given by the basic control surveys. Of course, the same object—harmony—could be attained by using geographic co-ordinates, but these may not be convenient to use in local projects. The State systems provide for harmony without losing the advantages of plane-surveying methods.

It would be ridiculous to use figures expressing latitude and longitude in setting column foundations for structures or for calculating volumes in earth-work. It would be equally ridiculous to struggle with plane rectangular co-ordinates, referred to the State systems, in plotting a map of a continent or in computing a far-reaching arc of triangulation. Nevertheless, the standard State co-ordinate systems can hold, in rigid relationship with one another, surveys that differ in extent and scope even as widely as these.

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DISCUSSIONS

PRESSURES BENEATH A SPREAD FOUNDATION

Discussion

BY MESSRS. ROBERT G. HENNES, T. A. MIDDLEBROOKS, AND A. A. EREMIN

ROBERT G. HENNES,[35] ASSOC. M. AM. SOC. C. E. (by letter).[35a]—This contribution by Professor Krynine to the growing literature on soil mechanics is of more than usual interest, not only because of the excellence of material and presentation, but also because the division of his paper into two parts illustrates rather aptly an important gap in the existing knowledge of this branch of the subject. Part I deals with the determination of numerical stress values in an ideal material under given conditions of loading, which is assumed quite generally to apply with acceptable accuracy to the case of stress distribution in a deeply underlying soil stratum. Part II deals in a more general sense with stress distribution over the surface of contact of the structure with the earth. For the average structure the stress distribution in the intermediate "disturbed zone" is of at least equal importance, but this region constitutes also an "unexplored zone," which needs more investigation before a satisfactory theory is available to the profession. In the meantime, the engineer must resort to the statistical, as opposed to the scientific, approach in dealing with what Professor Krynine terms "contact settlement."

Those who frequently have occasion to determine stress distribution in the subsoil should welcome the graphical methods presented in Part I. Others, who only seldom undertake this task, can profitably study at least that part of the author's method which presents Kögler and Scheidig's use of annular elements, especially when confronted with non-uniform loads, or with loaded areas of irregular shape.

It is natural, but unfortunate, that most valuable contributions to the science of soil mechanics seem to widen progressively the gap between the

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NOTE.—The paper by D. P. Krynine, M. Am. Soc. C. E., was published in April, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1937, by Messrs. O. K. Fröhlich, Donald W. Taylor, and Jacob Feld; and October, 1937, by Messrs. G. P. Tschebotareff, and A. Hrennikoff.

[35] Asst. Prof., Civ. Eng., Dept. of Civ. Eng., Univ. of Washington, Seattle, Wash.

[35a] Received by the Secretary October 11, 1937.

specialist and the average civil engineer. It is the impression of the writer that many engineers without formal education in soil mechanics would read this, and similar papers, more profitably if the fundamental assumptions were treated from the physical as well as from the mathematical viewpoint.

Consider, for example, the case of a uniform load placed on the horizontal surface of a large supporting body. If the load could be transmitted through this material without spread, the resulting situation would be as shown in the free body diagram of Fig. 18, the entire load, *W*, being carried by the prism *ABCD*, which would, consequently, be shortened. If the material possessed any shearing strength, however, this compression would set up shearing stresses along the surfaces, *AD* and *BC*, thus transmitting part of the load to the adjacent regions, in violation of the assumptions. If the material had no shearing strength, the underlying material would flow from under the load until the vertical stress across any horizontal plane became equalized, and the body would float, as in Fig. 19. If the material possessed some shearing strength, but less than the maximum stress imposed, there would be flow until the maximum shearing stresses were relieved to a point within the capacity of the material to withstand them. The quantity of flow depends, in part, upon the location of the over-stressed material. If it cannot move without displacing a considerable quantity of adjacent material, the flow may be limited to the relatively small amount required to compress the adjacent under-stressed soil sufficiently to transfer to it the excess load. Because this quantity depends upon the compressibility of the material, the computation of stress distribution is greatly simplified if it is first assumed that the stresses nowhere exceed the shearing strength of the material.

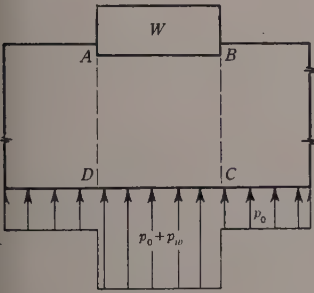

FIG. 18.

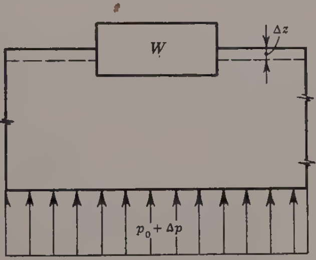

FIG. 19.

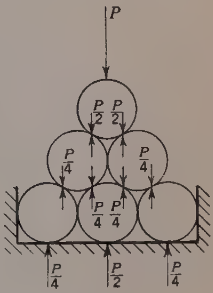

FIG. 20.

Next, consider a pile of incompressible rollers arranged as in Fig. 20. It is obvious that the load, *P*, would be divided equally between the two rollers in the second layer, and that in the third layer the middle roller would be subjected to a load of $0.5\,P$ while the two end rollers would each carry a load of $0.25\,P$, horizontal stress components being disregarded. If the same stress distribution were assumed to occur in a pile of elastic rollers, the application of the load, *P*, would result in a compression of each roller, and in the third layer the middle roller would experience twice the vertical compression of the end rollers, and hence lose contact with the overlying rollers which transmit the load. Since this is contradictory, it seems that for elastic materials the normal stress distribution on a horizontal plane will be more uniform than that represented by the example of the inelastic rollers. An intermediate case would be that of a material which, like soil, offered relatively more resistance to compression at higher temperatures. It should be noted further that, in the foregoing example, lateral forces have been neglected. As the end rollers experienced lateral displacements under stress, the middle roller would regain its proper share of the load, as the overlying rollers moved into more stable positions with equal bearing on their supports. Since vertical stress distribution is thus seen to be a function of lateral strain, it follows that divergence between the stress-strain relationships along horizontal and vertical axes would affect that nature of normal stress distribution. The general problem is further simplified, therefore, by assuming that the material has the same physical properties in all directions, and that it is a solid homogeneous mass.

In general, the foregoing assumptions are those of Boussinesq in determining the stress at a point in a semi-infinite, elastically isotropic solid, loaded by a single concentrated normal force applied at a point on the horizontal boundary plane.

With the foregoing concepts, and with a knowledge of Boussinesq's assumptions, one can visualize, in a general way, the application of this type of equation to lightly stressed soil strata, especially when the concentration factor is included. If those concepts are approximately correct, however, it is difficult to understand how the concentration factor can be made to cover the situation that arises when the shearing strength of the soil is exceeded at one or more points, involving such factors as the flow properties and the compressibility of the material. Yet this is the situation that commonly exists in the disturbed zone, at least under the edge of the footing where shearing stress is maximum. Even the lightest surface load, therefore, is likely to cause a trace of flow at the edge of the footing. If the footing is perfectly flexible, the flow continues until the edge of the footing has settled to such a depth that the weight of the adjacent earth has sufficiently served to relieve the intensity of the maximum shearing stress, or to increase the frictional resistance to flow. If the footing is perfectly rigid, the edges cannot settle independently; thus incipient failure of the soil at the edges is accompanied by transfer of some of the load to the central part of the footing, and the condition of uniform loading no longer holds. If this view is acceptable, Fig. 11(*a*) and Fig. 11(*b*) generally do not hold for surface loadings, whereas for footings at considerable depth, the effect of confinement might give even clay sufficient rigidity to correspond to Fig. 11(*c*) and Fig. 11(*d*). It would still be possible, of course, for the intensity of the soil reaction at the center of the footing to be relieved by the actual settlement of the soil, as in the case of consolidation of underlying clay strata.

T. A. MIDDLEBROOKS,[36] ASSOC. M. AM. SOC. C. E. (by letter).[36a]—The factors that influence the stresses in earth masses are treated most completely in this paper, which reveals clearly the fact that little is known about the true distribution of stresses in a foundation, especially where there is a wide difference between the rigidity of the structure and the foundation material. A quick realization of this fact by the Engineering Profession in general will be most helpful in speeding the accumulation of badly needed field data. The author's "reduced area" method of determining pressures under structures is a decided improvement over the usual procedure, and should meet with widespread usage.

Recognition of the influence of the rigidity of structures and earth masses on the stress distribution is of major importance. Professor Krynine has given a thorough presentation of the effect of this influence and he rightly states that the degree of rigidity in actual structures will be some value between the two extreme limits. The writer agrees with the statement that a clay can be considered as a non-rigid mass; but, on the other hand, he believes that a compact sand will act as a rigid mass only when it is supported by non-yielding strata. In the case of a footing resting on a compact sand stratum which, in turn, is supported by a non-rigid clay stratum, the sand, in this case, could not be correctly considered as a rigid mass. The same is true of a sand dam resting on a clay foundation. In cases where the clay is underlaid, as well as overlaid, with sand, the top sand should not be treated as a rigid mass, although the sand beneath it should be considered rigid. Working on this basis, the writer has been able to obtain some close checks on the stability of earth fills resting on clay in the foundation.

Interrelation of stresses in the structure and foundation are of utmost importance. The stresses in an earth fill have a marked effect upon the stress in the foundation. This fact has been ignored by numerous investigators in an attempt to devise a simple formula or method for analyzing the stress in a foundation. The writer has found that it is essential, in the design of earth embankments, that the fill and foundation be considered as a whole in determining the stresses in the foundation.

Considerable errors will occur in the computed foundation stresses if the stress in the structure is not taken into consideration, as would be the case if it is assumed to be perfectly rigid. It is evident that the greatest error in these stresses occurs when the actual rigidity of the structures and the foundations is approximately the same. Errors are also introduced if it is assumed that the vertical load on the upper boundary of the foundation material is equal to the weight of the material directly above it at every point, and that the shearing stress along this upper boundary is zero. It is only logical that "dam builders" should be first to recognize the importance of these principles, since the difference in the rigidity of the fill and foundation of a dam is much smaller than it would be for a footing on the same foundation.

In making use of computed foundation stresses, does the author consider satisfactory the generally accepted method of determining the probable settlement of a structure by computations based on the use of the vertical forces and the change in volume of the compressible strata directly beneath the loaded area only? In order to determine accurately the probable settlement beneath a footing, the volume change from consolidation throughout the entire pressure bulb should be used. Furthermore, the maximum principal stresses, rather than vertical stresses, should be used in computing the consolidation. This method will not be as simple to use as the generally accepted method, but too much simplification of a complex problem can easily lead to undesirable errors in the results. Therefore, since the engineer is attempting to determine the stresses more accurately, he is also justified in using a more nearly correct method of computing the consolidation.

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[36] Senior Engr., Chf., Soil Eng. and Fill Control Div., Fort Peck Dam, Fort Peck, Mont.

[36a] Received by the Secretary October 21, 1937.

A. A. EREMIN,[37] ASSOC. M. AM. SOC. C. E. (by letter).[37a]—A most interesting method of graphical computation of foundation stresses under spread footings is developed in this paper. It is simple and helps to visualize the distribution of foundation pressures. Professor Krynine's graphical construction may also be applied in the computation of foundation stresses with equations developed by Messrs. F. Kögler and A. Scheidig.[38] Using the author's notation, the vertical foundation pressure, p_z, at depth, z, as expressed by Messrs. Kögler and Scheidig is:

$$p_z = \frac{3\,P}{2\,\pi\,z^2}\,\frac{(\cos\alpha - \cot\phi\,\sin\alpha)\cos^4\alpha}{1 - \cos\phi} \quad \ldots\ldots\ldots\ldots\ldots (38)$$

in which ϕ is the angle of repose of the foundation material. Considering the principle of superposition of loads, the foundation pressures due to the load on an elementary area, dA, from Equation (38) are,

$$p_z = p\,\frac{3}{2\,\pi\,z^2}\sum\frac{dA(\cos\alpha - \cot\phi\,\sin\alpha)\cos^4\alpha}{1 - \cos\phi} \quad \ldots\ldots\ldots\ldots (39)$$

The summation of the last term in Equation (39) may be determined by a graphical construction similar to that shown in Fig. 2.

It is interesting to note that instead, of the term, n, used by the author in the equations for foundation stresses, Messrs. Kögler and Scheidig used the angle of repose, ϕ, to express the physical qualities of the foundation material.

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[37] Associate Bridge Engr., Bridge Dept., Div. of State Highways, Sacramento, Calif.

[37a] Received by the Secretary October 23, 1937.

[38] *Bautechnik*, 1929, p. 268.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

HYDRAULIC TESTS ON THE SPILLWAY OF THE MADDEN DAM

Discussion

BY F. W. EDWARDS, JUN. AM. SOC. C. E.

F. W. EDWARDS,[17] JUN. AM. SOC. C. E. (by letter).[17a]—The tests reported in this paper, conducted as they were in 1931, have furnished a pattern for numerous later experiments on similar structures.[18] In addition, the paper describes the preparations made for comparative tests on the prototype. Unfortunately, as repeatedly emphasized by the author, the spillway discharges to date have been insufficient in magnitude to make direct comparisons with the model. Extrapolations in studies of this nature are very unsatisfactory. However, the paper should serve to stimulate interest in obtaining data, and thus insure complete measurements on the prototype when the opportunity is presented.

Model Tests.—It is interesting to review the conclusions reached from the model tests of Madden Dam in the light of the results obtained from the model tests of the Conchas Dam in New Mexico,[19] conducted at the U. S. Waterways Experiment Station in 1936.

Conclusions (1), (5), (6), (7), and (10), in Part I of the paper, were entirely substantiated by the Conchas Dam tests. Conclusions (8) and (9), when qualified slightly, were also substantiated by the Conchas Dam tests.

The tests at Conchas Dam indicated that the apron could be shortened by use of a sill (see Conclusion (8)) but, in addition, they indicated the advantages of an end sill when the apron of the stilling-basin extended down stream beyond the point of full recovery for the hydraulic jump. Several types of end sills, including the sloped-face type, were tested, and

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NOTE.—The paper by Richard R. Randolph, Jr., Esq., was published in May, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1937, by Messrs. Ettore Scimemi, and J. C. Stevens.

[17] Associate Engr., U. S. Engr. Office, Second New Orleans Dist., New Orleans, La.

[17a] Received by the Secretary October 2, 1937.

[18] "Hydraulic Model Tests, Madden Dam," *Engineering News-Record*, Vol. 109, 1932, p. 42.

[19] "Model Tests of Conchas Dam Stilling Basin," U. S. Waterways Experiment Station *Technical Memorandum No. 105–1*, July 21, 1936, addressed to The Dist. Engr., U. S. Engr. Office, Tucumcari, N. Mex.

a stepped sill was selected as the best of those investigated. The essential factor, therefore, is the sill rather than the specific type of sill.

No doubt, the plan suggested in Conclusion (9) produces the very best conditions, but the tests for Conchas Dam indicated that where the stilling-basin is confined within a relatively narrow canyon, the side walls could be sloped off on the down-stream end without seriously interfering with the jump action. As far as stream-bed velocities and erosion down stream were concerned, the essential feature was an adequate apron having an end sill.

Conclusions (2), (3), and (4) were not factors in the design of Conchas Dam, where a horizontal apron was feasible.

The final conclusion, (11), listed by the author in Part I, incorporates more than its first sentence would imply. Although the writer agrees thoroughly with the first part of the paragraph, the remainder appears to delve too much into matter of opinion instead of presenting a definite conclusion based upon the model tests.

There are two schools of thought regarding baffles. One believes that baffles are absolutely essential in combination with the hydraulic jump to help dissipate, sufficiently, the excess energy in the over-fall from a high dam. The other school believes, like the author, that baffles are actually dangerous. It appears that a compromise might be effective in some cases. This was the attitude taken at Conchas Dam where one row of baffles, placed far enough down stream to avoid excessive impact, was used. The stilling-basin selected from the model tests performed satisfactorily when equipped with an end sill alone, but the one row of baffle-blocks was added principally to improve the performance of the basin when the conduits were discharging alone. The baffles definitely improved the action of the jump for high spillway discharges as well, and were considered an additional factor of safety in the design. By placing the baffles relatively far down stream, full advantage was taken of the impact of water on water before giving additional aid with the baffles.

Prototype Tests.—The agreement between corresponding nappes and pressures over the crest of the spillway for the model and its prototype appears to be highly satisfactory. The indicated energy gradients also appear to be in satisfactory agreement. However, the implication regarding coefficients of discharge for the spillway (see Fig. 15) should not be overlooked. Judging from the results of others, whose methods of measuring discharge were superior to those described for Madden Dam, one is inclined to accept the model results at least for the maximum heads. No doubt, viscosity and surface tension do affect coefficients of discharge for very small heads in models, but it seems unlikely that the effect would be great for heads of 0.45 ft (32 ft in Nature), the maximum head on the model of Madden Dam.

It would be extremely interesting and worth while to the Engineering Profession if measurements could be made upon the spillway similar to those made by Albion Davis and the late Floyd A. Nagler, Members, Am. Soc.

C. E., on the Keokuk Dam,[20] and to those made by Louis G. Puls, Assoc. M. Am. Soc. C. E., on the Wilson Dam.[21] These measurements would be especially valuable because of the comparison that could be made with the relatively small model.

In discussing friction loss at the toe of the dam, the author states that, "the difference between the velocity head as indicated by the Pitot tube readings and the theoretic velocity head (which is the difference between water surfaces) was taken as the loss in head due to friction on the overflowing sheet of water." The statement in parenthesis indicates that the correct value for the loss of head was used; but it is incorrect to state that the theoretical velocity head equals the difference in water surfaces, or that the head loss is the difference between the measured and the theoretical velocity heads. To obtain the true loss in head, it is necessary to take the difference between the theoretical energy line (theoretical depth plus theoretical velocity head) and the actual energy line (actual depth plus actual velocity head). The fact that the theoretical depth is different from the actual depth makes the author's statement incorrect.

Although the abscissas in Fig. 17 are entitled, "Values of h_f, Head Loss at Toe of Dam of H Height," it appears that the numerical values shown are for the Madden Dam, of specific height.

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[20] "Experiments on Discharge Over Spillways and Models, Keokuk Dam," *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 777.

[21] "Spillway Discharge Capacity of Wilson Dam," *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 323.

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DISCUSSIONS

PRE-STRESSED REINFORCED CONCRETE AND ITS POSSIBILITIES FOR BRIDGE CONSTRUCTION

Discussion

By Messrs. Charles S. Whitney, and Karl W. Lemcke

Charles S. Whitney,[19] M. Am. Soc. C. E. (by letter).[19a]—The pre-stressing of reinforcing steel is of considerable advantage in the case of some special uses of concrete and has very interesting possibilities. The writer believes that Mr. Rosov is over-enthusiastic regarding the general application of the method to concrete structures. It may be that the author emphasizes its theoretical advantages without full consideration of the practical limitations.

Concrete is a very imperfectly elastic material, and any attempt to estimate the actual stresses in a pre-stressed member under loading conditions is certain to result in a very rough approximation. Pre-stressing improves the behavior of concrete under load by reducing the cracking, but because of the large strains which occur before failure, pre-stressing will have practically no effect on the ultimate strength of the member. Therefore any increase in allowable load due to pre-stressing will reduce the factor of safety against ultimate failure. No doubt this is warranted in some types of structures because of an increase in the useful strength limit.

The difference between the two slabs indicated in Fig. 5 cannot be justified by pre-stressing the steel, but is largely due to the inadequacy of the conventional design method used.

Applying the formulas recently proposed by the writer,[20] and assuming the concrete cylinder strength, $f_c' = 2\,000$ lb per sq in., and the steel elastic limit, $f_s = 40\,000$ lb per sq in., the ultimate bending strength of the conventional

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Note.—The paper by Ivan A. Rosov, M. Am. Soc. C. E., was published in September, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[19] Cons. Engr., Milwaukee, Wis.

[19a] Received by the Secretary October 2, 1937.

[20] "Design of Reinforced Concrete Members Under Flexure or Combined Flexure and Direct Compression," by Charles S. Whitney, *Journal*, Am. Concrete Inst., March–April and September–October, 1937, Equations (7), (8), and (6), respectively.

design (Fig. 5*a*) per foot width of slab, expressed by[20]:

$$M = b\, d^2\, p\, f_s \left(1 - \frac{p\, f_s}{1.7\, f_c'}\right) \tag{43}$$

is equal to 1 945 000 in-lb. For balanced reinforcement[20]:

$$p = 0.456\, \frac{f_c'}{f_s} \tag{44}$$

or $p = 0.0228$; and the ultimate moment, as expressed by[20]:

$$M = \frac{b\, d^2\, f_c'}{3} \tag{45}$$

will be $12 \times 25 \times 25 \times \frac{2\,000}{3} = 5\,000\,000$ in-lb. The author's conventional design, therefore, has a factor of safety of $\frac{1\,945\,000}{12 \times 61\,400} = 2.64$ against steel failure and a factor of safety of $\frac{5\,000\,000}{12 \times 61\,400} = 6.8$ against concrete failure.

According to the author's statements an allowable unit stress of 35 000 lb per sq in. in the steel for the pre-stressed design would require an elastic limit of about 90 000 lb per sq in. Assuming this value of f_s and applying the writer's formulas to the design shown in Fig. 5(*b*), the percentage of steel required for balanced reinforcement would be (see Equation (44)), $0.456\, \frac{2\,000}{90\,000} = 0.0101$. The author's design provides: $p = \frac{1.65}{12 \times 13.5} = 0.0102$.

The ultimate strength by Equation (45) is $12 \times 13.5 \times 13.5 \times \frac{2\,000}{3} = 1\,458\,000$ lb per sq in. The factor of safety of the author's design against ultimate failure is, therefore, $\frac{1\,458\,000}{12 \times 44\,600} = 2.72$, whether or not the steel is pre-stressed. The pre-stressing will reduce the amount of cracking in the concrete, but that can also be reduced by using more steel, which could then have a lower yield point and which would provide lower bond stresses. For ordinary structures the pre-stressing procedure does not appear advantageous.

Karl W. Lemcke,[21] Assoc. M. Am. Soc. C. E. (by letter).[21a]—Pre-stressed reinforced concrete is somewhat similar to the jacking of moments into truss members in order to nullify or greatly decrease objectionable secondary stresses. In each case the capacity of the member involved is increased without adding to its section. The author has presented an interesting and timely paper.

Although considerable progress has been made, the writer believes that reinforced concrete as a structural material is capable of much further development. While American engineers are generally in the forefront of progress, in the matter of conceiving long-span concrete bridges, they seem to be behind European engineers. As against the 460-ft reinforced concrete arch span of the George Westinghouse Bridge, at Pittsburgh, Pa., the French have the 612-ft arches of the Plougastel Bridge, near Brest. Freyssinet, the designer of the latter structure, has very ably stated the reasons why greater progress has not been made in long-span concrete construction.[22]

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[21] With Robinson & Steinman, New York, N. Y.

[21a] Received by the Secretary October 26, 1937.

Referring to pre-stressed concrete design, Mr. Rosov states in effect that, for long spans an I-shaped girder should be used, but that in conventional concrete design such girders are not used because the bottom flange would be useless, due to its tendency to crack. One such concrete girder bridge of conventional design and I-section was designed and built across Salt Creek in Humboldt County, California.[23] Reinforced concrete was chosen in preference to structural steel because it was felt that, due to the heavy fogs and moisture at the site, structural steel would be subject to great corrosion. The concrete design was also cheaper. The bridge was a two-span continuous one, 284 ft long. The girders were 12 ft deep, the top flanges, 2 ft wide, the bottom flanges, 1.5 ft wide, and the web varied in thickness from 7 in. to 18 in. As this structure was built about 1920 it would be interesting to know how it has withstood the elements, and whether the bottom flanges have cracked.

The author gives some interesting comparisons of designs for a highway loading and a 25-ft clear span, referring also to the conventional designs of the Delaware, Lackawanna and Western Railroad, as of 1926. In this connection the reinforced concrete grade-crossing designs of the Canadian National Railways might be of interest.[24] Among other types, single spans as long as 60 ft, both precast and poured in place, have been developed to carry Cooper's E-60 live load. It is stated that economies of from 30% to 55% are shown by these structures over those built in the past; if designed of pre-stressed reinforced concrete they would probably show a still greater saving.

It seems to the writer that the 10-in. slab thickness mentioned for a 25-ft clear span is rather shallow even for pre-stressed reinforced concrete construction, and might produce excessive deflections; also that 2 000-lb to 2 500-lb concrete was a rather low grade to use, although the author may have taken it purposely.

The advantages of having no bends in the steel reinforcement would probably be offset by the increased cost of forms, due to having to increase the depth or width of the members.

On jobs that are poured in the field the pre-stressing of the reinforcement might be quite expensive due to the high wages of labor, but this cost would probably be reduced materially after the operation had been standardized.

In conclusion, the writer believes that Mr. Rosov's paper should call to the attention of the profession the advantages and economies of pre-stressed reinforced concrete, and should also further its development.

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[22] *Civil Engineering*, February, 1932, p. 96.

[23] *Engineering News-Record*, February 26, 1920, p. 427.

[24] *Loc. cit.*, July 19, 1934, p. 65.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRACTICAL APPLICATION OF SOIL MECHANICS A SYMPOSIUM

Discussion

BY MESSRS. S. C. HOLLISTER, T. T. KNAPPEN, AND L. F. HARZA

S. C. HOLLISTER,[47] M. AM. SOC. C. E. (by letter).[47a]—Mr. Hough is to be congratulated on the development of the model tests, reported in this paper, by means of which he was able to determine qualitatively the nature of settlement of the rock fill and the underlying clay bed.

Several points occur to the writer in regard to the determination of the safety of such a construction. The first of these relates to the fact that the rupture surface lies almost entirely within the bed material. This condition is characteristic when the material in the embankment has a high shear strength in comparison to the shear strength of the foundation material.

The second point refers to the stability of such a structure. When the rock fill has been completed and movement of the clay ceases, there exists at that time a factor of safety of one. This means that slight additional loading of the fill would cause a resumption of flow in the clay bed. If the rock fill came to rest in part upon the rock stratum below the clay, stability would be greatly increased. Furthermore, if the mud wave were counterweighted, stability again would be increased.

The "heart-shaped" failure surface, now that it is noticeable by the tests, is discernible from the theory of elasticity. A complete solution of the problem shown in Fig. 38, in which the foundation bed is loaded not only vertically but tangentially, is much needed.

THEODORE T. KNAPPEN,[48] M. AM. SOC. C. E. (by letter).[48a]—In the light of his personal experience with levee construction in this region, the writer desires to congratulate Mr. Buchanan on his accurate presentation of the

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NOTE.—This Symposium was presented at the meeting of the Soils Mechanics and Foundations Division, at Boston, Mass., October 7, 1937, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1937, by the members of the Committee of the Society on Earths and Foundations.

[47] Dean, Coll. of Eng., Cornell Univ., Ithaca, N. Y.

[47a] Received by the Secretary October 11, 1937.

[48] Associate, Parsons, Klapp, Brinckerhoff, and Douglas, New York, N. Y.

[48a] Received by the Secretary October 19, 1937.

Mississippi River levee problem. The soundness of the old design methods is amply demonstrated by the success of the Mississippi levee system in the 1937 flood, just as the soundness of the work of the Old Master Builders is demonstrated by the many ancient buildings that are standing to-day. Precisely as modern structural design has modified building practice, so will the rational methods of soil mechanics, under the leadership and guidance of experts, modify levee construction and gradually transform it from an art to a science.

Although it is true that most of the work on main line levees has been completed, there is still much to be done to provide an adequate factor of safety at the weak places in the existing system and to construct the tributary extensions. It is hoped that there will be sufficient infiltration of the principles expounded by Mr. Buchanan into the various design staffs to insure the application of rational design methods to the work yet to be done. Their best application is in the hands of these engineers who have, for years, carried on the design activities on the river. Familiar as they are with the ground conditions, they can apply the rational design methods with the conservatism that is necessary where protection of life and property is involved.

The ideal condition would permit wide variations in design resulting in a uniform factor of safety. Practical conditions and limitations of knowledge and its dissemination make the attainment of this condition a remote object to be approached gradually. The writer's comments are directed at the elucidation of a few of the factors discussed with the aim of clarifying some of the basic principles involved.

First, examine the standard Mississippi River sections described as *A*, *B*, and *C* in Table 1. The *A* section is for a material containing 75% or more of clay, defined on the river as material finer than 0.005 mm. Although this section has never been discarded it was early learned by the field engineers that levees, even of moderate height (15 to 20 ft), built of material in this classification, were not stable with slopes as steep as 1 on 3. On the other hand, such materials are so impervious that seepage never reaches the down-stream slope except through pervious foundations. Actually, with such materials both slopes should be the same, assuming foundations of similar material.

Now, it will be noted that a river-side slope of 1 on 5 is provided for the *C* sections. On analysis most of these materials will be stable with a slope of about 1 on 2, but in actual experience wave wash has proved so troublesome with the usual fine cohesionless material of this class that this slope is a practical necessity unless slope paving is used. Most materials in this class are sufficiently pervious to establish the normal seepage gradient during a flood. To provide stability without under-drainage the down-stream slope is, properly, quite flat. Greater economy and stability might be obtained by the use of drainage provisions, largely through selection of available materials.

Most of the levees are actually of the intermediate or *B* classification and the levee dimensions indicated are usually more than adequate; but they have limited or inadequate safety factors in a few cases, particularly as one limit or the other is approached.

Mr. Buchanan has discussed the method used in foundation design where a comparison of stress versus strength is used. He has indicated the limitations of this method. More emphasis might well be placed on the photo-elastic method of stress analyses in particularly difficult cases.

In analyzing embankment stability Mr. Buchanan has used the sliding-surface method. The writer is inclined to believe that the inaccuracies of this method are such that it would be better to develop the stress-strength relationship in a manner similar to foundation design practice. The photo-elastic method offers one approach to this problem. Incidentally, where embankment and foundation materials are both of questionable strength the two analyses should not be separated but should be treated as one problem.

It has been the practice for a number of years to provide for the installation of settlement gages (locally known as "crow's feet") as a means of measuring foundation settlement for payment purposes. These gages offer an excellent means of detecting incipient foundation failure. Where foundation investigations have been made in advance an estimate of the rate of consolidation can be made and compared with actual settlement. If excessive settlement develops, incipient failure is indicated. In such a case immediate revision of the design to prevent failure should be made. Even where a foundation analysis has not been made the behavior of these gages offers a good indication of trouble to an experienced inspector.

A common indication of incipient foundation failure is an upheaval of the ground surface beyond the embankment toes. Prior to failure this movement usually is small; but it is frequently visible to the eye and can be detected in most cases by instrumental measurements. By the establishment of hubs in the critical reaches in advance of construction a check may be maintained on the foundation condition. Likewise, embankment slides are indicated in advance by the bulging of embankment slopes. Instrumental checks can detect such conditions in sufficient time to permit corrective measures to be taken.

It is realized that many of the construction methods permitted on levee work do not lend themselves to the application of these methods of detection of probable failures. Some limitation on the method of construction in such cases may be advisable. Based on present-day hauling methods and costs there is no reason to believe that these limitations will result in materially increased costs.

With respect to the discussion of levee construction methods in the paper, it might be argued that an increase in cost of 4 cts to 6 cts per cu yd for compaction would be more than offset by economies in section design. However, it should be remembered that most of the levee work was executed before present-day knowledge of soil mechanics had advanced to the point where it could evaluate the improvement. It is the writer's opinion that the general solution was sound for the time that it was made, but that in the future it should be modified in the light of recently acquired knowledge.

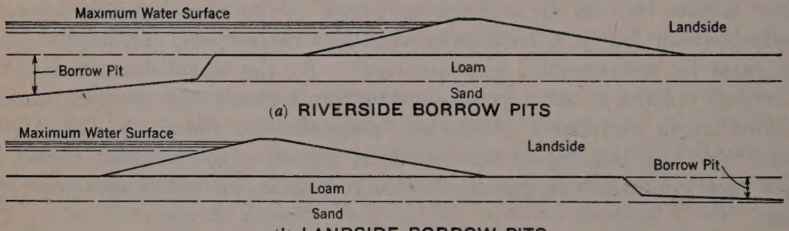
One of the most troublesome problems in levee planning is borrow-pit location. Mr. Buchanan points out that considerations of economy require that the material be obtained opposite, and as close as possible to, the proposed levee. He further states (see heading, "The Design of a Levee Unit") that,

> "It is important that no borrow-pits or excavations be made adjacent to the levees on the land-side to expose pervious strata extending through to the

river side. Exposure of such strata invites trouble from increased underseepage, certainly; and from destruction of the levee structure due to piping, possibly. Although in some instances, borrow-pits have been made on the land side, they are the source of never-ending trouble."

This policy has resulted in the use of river-side pits almost exclusively in recent years. These pits are separated generally from the levee toe by a 40-ft berm. At the edge of the berm they are 3 ft deep and deepened from 2 ft to 4 ft in every hundred depending on the specification in each case. Below the mouth of the Red River slopes of 10 ft in 100 are permitted. One of the greatest fields for improvement in levee design lies in the application of the principles of soil mechanics to borrow-pit location and design in connection with the hydraulic design of levees.

The Mississippi levees are built on, and of, alluvial soils. This means that almost every combination of sands, loams, and clays may be encountered. In some cases deep foundations of relatively impervious loam or clay exist. In such cases the only restrictions on borrow-pit depth need be stability and future enlargement considerations. More frequently the foundations are stratified, making a careful analysis of the problem most important. Often there is a layer of loam overlying a deep sand stratum as in Fig. 56(*a*). In this case the river-side borrow-pit cuts through the loam exposing the sand, which may be one hundred times as pervious as the loam. Here the head acting through the pervious sand on the loam stratum causes boils or piping at the land-side toe of the levee. This offers an explanation for many of the worst troubles on the levee system. In time the borrow-pit will silt up in the small annual floods and the trouble tends to become less in succeeding large floods.



(*a*) RIVERSIDE BORROW PITS

(*b*) LANDSIDE BORROW PITS

Fig. 56.—Typical Levee Cross-Section

Now, reverse this condition and put the borrow-pit on the land side as in Fig. 56(*b*), taking care to excavate to the sand. The loss of head then occurs to a large extent in downward flow on the river side where it can do no harm. In this case the land-side pit is preferable. Now, land-side specifications only permit a deepening of 1 ft in 100; so the pit in this case would have a thin covering of loam over the sand. The result would be boils in the pit which would look bad although they would not be serious.

There are, of course, many other combinations. One would be the case of the pervious sand foundation overlying an impervious stratum like the case cited at Mound's Landing, where the treatment of a clay blanket extending to the impervious stratum was excellent. There are many other known weak spots in the system where a soil mechanics analysis should develop a solution whether it be by cut-off, blankets, or drainage. The important consideration is that Mr. Buchanan's studies should be extended to the solution of these problems. The present rigid borrow-pit requirements should be modified by individual design studies based on the exploration borings described in the paper.

L. F. HARZA,[49] M. AM. SOC. C. E. (by letter).[49a]—The need of a reliable basis of stability analysis for foundations under earth embankments is very great indeed. Much has been written and much research has been conducted applicable to selection of suitable embankment materials and their compaction to optimum density and "stability."

All this refinement in the embankment itself, however, amounts to little unless the foundation or underground is equally stable, or unless means exist for interpreting laboratory tests of foundation material into degree of stability under conditions of loading. Experience at Zanesville, Ohio, and elsewhere clearly indicates that knowledge and technique in building artificial embankments has progressed far beyond knowledge of whether a foundation will properly support the embankment when built. In general, the foundation must be accepted as it exists, and it is likely to be by far the weaker and more uncertain element of the structure, subject to much greater settlement and seepage.

Any foundation settlement which tends to cause cracks or fissures in the superstructure invites failure in the embankment. Data and methods of analysis should eventually become available for indicating the degree of flexibility of embankment sections and profiles for accommodating themselves to foundation settlement without cracking and for predicting such settlement from laboratory tests. If the Haines method, recommended by Mr. Hough, tends in this direction it should be welcomed.

His proposal to build dams of normal section, of cohesionless material, on foundations too soft to support the load without flow, in lieu of dams of such flat slopes as to avoid flow and thus limit distortion to vertical compression, is indeed very interesting. However, what might seem to be an obviously feasible, economical, and safe procedure at the Passamaquoddy project might, with the very unique conditions which seem to justify it there, never find another similar application.

It is scarcely conceivable that the new fill, in adjusting itself to the outflow of the soft foundation, could settle without cracks and fissures. Moreover, where the new fill is required to sag in profile, the bottom also tending to spread with outflow of soft foundation, invisible fissures in the bottom are probable, resulting in arching cavities in the fill. No cohesionless material is so fluid as to readjust itself without initial arching voids.

In the Passamaquoddy submerged fill, water could be allowed to flow back and forth through the pervious fill with each change of the tide until the structure is thoroughly settled and these cracks thus closed before applying the less pervious blanket. Failure at this stage could not occur. In a normal fill above water level, however, the final necessary readjustment of the fill to settlement could probably not be accomplished without filling the reservoir. Any distortion sufficient to accomplish such readjustment would seriously endanger the safety of the fill during the readjustment period if the reservoir were thus filled.

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[49] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

[49a] Received by the Secretary October 21, 1937.

To be successful, the scheme would seem to be dependent upon conditions in which anticipated settlement would be practically completed during construction. It is doubtful whether this could be done except in a submerged dam where water would be available for settlement without unbalanced pressure as at Passamaquoddy.

Corrections for Transactions: Change the denominator of Equation (6) to read: "$(1 + C_s^2) - (C_s - A_c)\,(C - B)$"; and, in the definition of r_w near the center of Fig. 22, change the last two words, "Core Material" to "Shell Materials."